Modelling meltwater drainage in the Paakitsoq region, western Greenland, and its response to 21st century climate change

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MPhil Polar Studies

June 2012

Abstract

Recent studies suggest that the Greenland Ice Sheet (GrIS) is thinning rapidly and contains sufficient water equivalent to raise global sea levels by ~7m. The dynamic response of ice sheets to rising air temperatures has been well documented, whereby increased surface meltwater accessing the ice sheet bed results in high subglacial water pressures that lubricate the bed and enhance basal motion. This could in turn drastically amplify the contribution of GrIS to sea-level rise (SLR). However, the inadequate formulation of ice dynamics in current ice-sheet models was identified as the largest source of uncertainty in SLR projections in the latest IPCC AR4 report. This thesis focuses on the supraglacial and subglacial hydrology of the Paakitsoq region in western Greenland to provide an insight into present-day dynamic behaviour of the ice sheet, and its potential response to climatic warming over the 21st century. Surface meltwater production and routing for 2005 are simulated using a surface mass balance model and a positive-degree day model, which in turn provides the input to a distributed, physically based model of subglacial drainage applied to Paakitsoq. The drainage model is then forced with future climate scenarios based on the IPCC's new framework of Representative Concentration Pathways (RCPs). The simulations produce a distinctive split in "marginal" and "inland" drainage behaviour at Paakitsoq, whereby the inland system is typified by low discharge magnitudes, high water pressures and small conduit CSAs, and the marginal system displays high discharge, mid to low pressures and higher conduit CSAs. Comparisons between measured and modelled proglacial discharge suggest that the appropriate *k* factor (the ratio of water pressure to ice overburden pressure) for modelling the study system is between 0.925 and 0.95. The most extreme climate scenario predicts a ~7^{o}C rise from 2010 to 2100, which will result in proglacial discharge increasing by the end of the 21st century to levels four times greater than today. However, significant inter-annual variability in melt production at Paakitsoq will probably lead to high-melt years whose peak subglacial discharges may be even more intense. This could cause considerable inland migration of channelised behaviour. The implications for future ice dynamics in the Paakitsoq region are considered.

Acknowledgements

I would like to thank my supervisor Neil Arnold [University of Cambridge] for his invaluable guidance throughout the whole project, from the conception of the study to the construction of its many models. Thanks are also due to Ian Willis [University of Cambridge] for his advice on aspects of the thesis focusing on future climate change. I am extremely grateful to Alison Banwell [University of Cambridge] for developing, formulating and running the supraglacial and subglacial models, and for interpolating the DEM. Without Alison's support, insight and endless enthusiasm, this thesis could not have been completed in time. The technical advice provided by Toby Benham [University of Cambridge], Valentina Radic [University of British Columbia], Cameron Rye [University of Oxford], James McMillan [University of Cambridge] and Philippa McClure [University of Cambridge] was much appreciated, and I thank them all for their time and admirable patience. I am very grateful to Emmanuel College for awarding me the Derek Brewer scholarship, which allowed me to study at the Scott Polar Research Institute. The camaraderie amongst all the MPhil students made this year all the more enjoyable. Finally, I am indebted to my family for their unending support and encouragement through some difficult times. I dedicate this this thesis to my mother, Isabelle Mayaud.

Declaration

I declare that this thesis is entirely my own work, except where otherwise acknowledged. The thesis does not exceed 20,000 words, excluding figures, tables, captions, contents, acknowledgements, abstract, references and appendices.

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June 2012

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1. Introduction

"Dynamical processes related to ice flow not included in current models but suggested by recent observations could increase the vulnerability of the ice sheets to warming, increasing future sea-level rise. Understanding of these processes is limited and there is no consensus on their magnitude."

- Fourth IPCC Assessment Report, 2007, Summary for Policy Makers -

Recent studies suggest that the Greenland Ice Sheet (GrIS) is thinning rapidly (Pritchard *et al.*, 2009) and contains sufficient water equivalent to raise global sea levels by ~7m (Sole *et al.,* 2011). The doubling of mass loss from GrIS over the last decade (Rignot & Kanagratnam, 2006; Khan *et al.,* 2010) is mainly attributable to the acceleration and retreat of outlet glaciers in western and southeast Greenland (Howat *et al.,* 2007), but several studies have highlighted the potential for increased surface melting to accelerate ice sheet flow. Zwally *et al.* (2002) and Joughin *et al.* (2008a) identified a direct link between seasonal surface melting and ice sliding velocity, which provides a mechanism for a rapid, large-scale dynamic response of GrIS to climatic warming. Hydrological studies of Greenland have increasingly focused on determining the nature and distribution of water flow on the surface and at the base of the ice sheet.

Given their relative inaccessibility, subglacial environments represent 'one of the last physical frontiers of glaciological research' (Hubbard, 2011:1). Knowledge of their characteristics is derived from a range of indirect methods, of which radio-echo sounding has proved extremely useful. There is general consensus (Shreve, 1972; Björnsson, 1982) that subglacial water flow is driven by gradients in hydraulic potential, and occurs through fast/channelized or slow/distributed systems located at the ice-bed interface (Fountain & Walder, 1998). Drainage structures evolve continually on a range of timescales in response to changing water inputs, influencing glacier geometry and changes in glacier flow dynamics (Kamb, 1987). Therefore, determining the processes linking Greenland's supraglacial and subglacial systems remains an important goal.

Field-based studies provide strong evidence that glacier and ice-sheet surface velocities respond dynamically to rising air temperature when meltwater reaches the ice-bed interface. This causes high subglacial water pressures and lubricates the glacier bed, enhancing basal motion (Iken and Bindschadler, 1986; Zwally *et al.*, 2002; Bingham *et al.,* 2003; Shepherd *et al.,* 2009; Bartholomew *et al.*, 2010; Clason *et al.,* 2012). Drainage of ponded meltwater (Das *et al.*, 2008) and the influx of supraglacial streams into crevasses or moulins (van der Veen, 2007) are likely sources for the reported dynamic forcing. If the observations of Zwally *et al.* (2002) and others are broadly applicable to the entire ice sheet, future warming might be expected to increase flow velocities, leading to ice sheet drawdown, thus increasing the area of the ice sheet subject to summer ablation (Parizek and Alley, 2004). However, there is ongoing debate about the long-term significance of such dynamic thinning and mass loss for GrIS (van de Wal *et al.,* 2008; Sundal *et al.,* 2011; Bartholomew *et al.,* 2011).

Subglacial processes contribute through complex feedback loops to changes in sea level, ocean circulation, and regional and global climate evolution. Yet the inadequate formulation of ice dynamics in ice-sheet models was identified as the largest source of uncertainty in the latest IPCC AR4 Report (Lemke *et al.,* 2007). This provides an impetus to improve our understanding of how supraglacial lake drainage events and subsequent subglacial water routing are likely to respond to increased surface meltwater production in a warming climate. The rest of this chapter reviews the theories describing subglacial drainage and the current understanding of the situation on GrIS, as well as the possible responses to future climate change. It concludes with the aims of this study, and the structure of the remainder of the thesis.

1.1 Subglacial drainage

1.1.1 Water inputs to the subglacial system

The subglacial system has four main inputs: surface, englacial, and basal melt, and subglacial groundwater (Sharp, 2005). Basal melt tends to be dominant in areas where surface air temperatures never rise above freezing point but ice temperatures reach pressure melting point (PMP) at the bed (e.g. Antarctica) or in areas of high geothermal heat flux (e.g. Iceland). On annual and shorter timescales, however, surface melt is usually the dominant source in temperate glaciers or wherever ice is at PMP. Supraglacial meltwater enters the englacial system via crevasses or moulins (Nienow & Hubbard, 2005). The magnitude and variability of surface water inputs depends on the density and exposure of the input sites as well as the nature of the melting surface (Nienow & Hubbard, 2005). The snowpack and firn can store considerable volumes of water, delaying melt-induced runoff and dampening diurnal and meteorologically driven variations in meltwater flux (Fountain, 1996). Ice surfaces have relatively little storage capacity, meaning that water inputs to the glacier bed from ice closely track the surface melt rate (Willis *et al.,* 2002), with water typically flowing 3-5 orders of magnitude faster than through snow/firn (Nienow & Hubbard, 2005). The proportion and spatial patterning of surface-derived water accessing the ice sheet bed have an important impact on the structure of the subglacial drainage system.

1.1.2 Types of subglacial drainage system

Two main types of subglacial drainage have been widely recognised, though each can assume a diversity of forms (Sharp, 2005): hydraulically connected, "distributed" water sheets, in which flow tends to be slow, and discrete channel elements, in which flow tends to be fast (Fountain & Walder, 1998). Although subglacial drainage beneath glaciers and ice sheets generally occurs on a spectrum between the two, both morphologies interact with each other (Hubbard *et al.,* 1995). Significant uncertainty surrounds their respective spatial and temporal distributions beneath GrIS.

Distributed drainage systems tend to consist of a widespread anastomosing network of linked cavities. These cavities form when ice slides over bed depressions and decouples from the bedrock (Fountain & Walder, 1998): the gap size between the protrusion and ice is then controlled by the opening rate due to sliding and by creep closure of the cavity roof (see Figure 1.1a; Schoof, 2010). Other distributed systems include braided canal networks (Walder & Fowler, 1994), transport through subglacial sediment (Fischer & Clarke, 1994), and a thin continuous film of water between the ice and bedrock (Weertman, 1972). The latter two morphologies, however, only play a minor role in water transport (Nye, 1976; Alley, 1989).

Channelised drainage systems can support much higher discharges, as they are more efficient than distributed systems. The primary components of channelised systems are large ice-walled conduits ("Röthlisberger", or R-channels; Röthlisberger, 1972): idealised semi-circular conduits in steady-state, whereby channel enlargement by wall melting is exactly balanced by creep closure (Figure 1.1b). Such assumptions predict an inverse relationship between the water flux through an R-channel and the pressure gradient that drives the water flow. In reality however, discharge is unlikely to be steady as it varies with surface melt rates on timescales shorter than those allowing channel cross-section adjustment (Sharp, 2005). Other channel types include conduits incised into subglacial bedrock ("Nye", or N-channels; Nye, 1973) and broad flat conduits formed in basal ice ("Hooke", or H-channels; Hock & Hooke, 1993).

Channelised systems have arborescent drainage pathways occupying only a small fraction of the bed (Figure 1.2a), offering little resistance to water flow and thus displaying large flow velocity changes in response to water flux variations (Nienow *et al.,* 1998). In contrast, distributed systems have anastomosing pathways (Figure 1.2b) that are hydraulically resistant, so that changes in water flux lead to large changes in flow cross section but little change in (generally low) flow velocity (Fountain & Walder, 1998). Theoretical analyses suggest that steady-state discharge-pressure relationships in channelised systems oppose those in linked-cavity networks (Kamb, 1987; Walder & Fowler, 1994). Since melt rates increase with water flux, water pressure decreases with increased discharge in conduits (Röthlisberger, 1972), causing water to flow from smaller channels into larger ones, favouring the formation of a few main channels at low pressure. In linked-cavity systems at high discharges, this relationship breaks down as ice melting becomes more important than water pressure in determining the size of ice cavities. In this case ice melt rates increase with discharge, resulting in a significant pressure drop. Therefore large, low-pressure cavities will grow by capturing drainage from small, high-pressure cavities, and can evolve into channels. The instability of linked cavity systems at high discharges thus provides a mechanism whereby subglacial drainage can "switch" to a more efficient mode (Kamb *et al.,* 1985).

Transitions between distributed and channelised systems occur both at the glacier scale (Hubbard *et al.,* 1995; Hubbard & Nienow, 1997; Mair *et al.,* 2002; Willis *et al.,* 2008) and at the ice sheet scale (Bartholomew *et al.,* 2010; Sole *et al.,* 2011). Such transitions are correlated with seasonal variations in ice flow velocity (e.g. Hubbard & Nienow, 1997; Bingham *et al.,* 2005, 2006) and ice dynamical instabilities (e.g. Nienow *et al.,* 1998; Kessler & Anderson, 2004). Theoretical simulations by Schoof (2010) suggest that the switch from a distributed to channelised system will occur when the opening rate exceeds the closing rate (Figure 1.3a) and when net effective pressure is at a minimum (Figure 1.3b). Recent numerical models (Flowers *et al.,* 2003; Pimentel & Flowers, 2010) have successfully simulated the dynamic switching between distributed and channelised drainage beneath ice caps and ice sheets. The evolution of the subglacial system has important implications for basal motion at the ice sheet scale (Bartholomew *et al.,* 2010), which will be explored further in section 1.2.2.

1.1.3 Routing of subglacial water flow

Bed and ice surface Digital Elevation Models (DEMs) derived from radio-echo sounding can be used to calculate hydraulic pressure potential fields (Shreve, 1972), which are the primary driving forces for subglacial water flow (Flowers & Clarke, 1999). Gradients in hydraulic potential can in turn be used to delineate drainage catchments and determine the location and size of drainage network elements (Pälli *et al.*, 2003). Assuming the bed substrate is impermeable, the flow direction will theoretically be downslope and perpendicular to the contours of pressure potential. Although the description of hydrological ice sheet drainage should include a formulation for englacial water routing (Shreve, 1972), it can be assumed that all meltwater reaches the bedrock and drains along the base of the ice sheet (Björnsson, 1982). This simplification is justified by many observational studies (Mottram *et al.,* 2009). Basal water flow can thus be determined by water pressure potential (Φ_b):

$$\Phi_b = \rho_w g Z_b + k \rho_i g (Z_s - Z_b)$$

(Equation 1)

Where ρ_w is water density (1000kgm^{-3}); ρ_i is ice density (917kgm^{-3}); g is acceleration due to gravity (9.81ms^{-1}); Z_b is the bed elevation (m); Z_s is surface elevation (m); k is a factor ranging from 0 (subglacial drainage system at atmospheric pressure) to 1 (subglacial drainage system at ice overburden pressure).

Drainage patterns are affected by overlying ice pressure variations, with basal water often supporting a substantial fraction of the ice overburden (i.e. high values of *k*). Shreve's equation can be rearranged to show that, after differentiation, the contribution of the ice surface gradient to the pressure potential gradient is a factor of $(\rho_w-\rho_i)/\ \rho_i \approx 10$ times that of gradients in the basal topography (Clarke, 2005). This implies that subglacial water can flow uphill out of glacier bed troughs if the pressure gradients exceed the elevation potential gradient. Several studies (e.g. Sharp *et al.,* 1993; Flowers & Clarke, 1999; Hagen *et al.,* 2000; Rippin *et al.,* 2003) show that a decrease in subglacial water pressure increases the role of bed topography on drainage routing, leading to a more dispersed, less efficient drainage system.

Whilst the assumption $\rho_w = \rho_i$ may be an acceptable approximation for winter conditions (when an efficient subglacial drainage system is not well developed), the *k* value varies greatly depending on local conditions and seasonal changes in meltwater input (Pälli *et al.*, 2003). *k* values as low as 0.5 have been shown to be appropriate for summer conditions in different glaciological contexts (e.g. Trapridge Glacier, Canada (Flowers *et al.,* 1999); Midre Lovénbreen, Spitsbergen (Rippin *et al.,* 2003); Brewster Glacier, New Zealand (Willis *et al.,* 2009)). It is also important to note the greater significance of conduit geometry and ice rheology than channel discharge in determining the basal system's water pressure (Röthlisberger, 1972; Ahlström *et al.,* 2005). Ahlström *et al.* (2002) have shown that the delineation of hydrological ice-sheet basins using high-resolution DEMs is very sensitive to changes in the *k* factor.

Shrevian analysis is of course purely geometric (Flowers & Clarke, 1999), and its assumption of uniform water-pressure distribution may not apply to all subglacial environments (Flowers *et al.,* 2003). Furthermore, it takes no account of the detailed subglacial physics appropriate for R-, N- or H-channels as proposed by Walder & Fowler (1994). Finally, the Shreve model does not include a formulation of feedbacks within the glacial system: by isolating the hydrological effects of surface and bed geometries that partly modulate its development, and by ignoring transitions in drainage mode over time, Shreve's model is an inevitable simplification of subglacial processes (Kessler & Anderson, 2004). Nevertheless, it remains a useful and informative overview of likely meltwater flowpaths beneath ice sheets.

1.1.4 Subglacial routing models

Present-day subglacial models have evolved significantly from the early analytical studies of Röthlisberger (1972) and Shreve (1972); they now seek to combine the location of subglacial flow with the nature of flow, and to formulate feedbacks between them in the context of a system composed of numerous glaciological elements. Particular challenges still remain in determining where surface water should reach the bed, and finding realistic ways of evolving the subglacial drainage system through time (Benn & Evans, 2010). The studies of Flowers and Clarke (2002a) and Arnold *et al.* (1998) represent two contrasting approaches to addressing these issues.

Flowers & Clarke (2002a) modelled the entire hydrology of a glacier by simulating the coupled evolution of surface, englacial, subglacial, and groundwater hydrological systems. Sensitivity tests revealed that subglacial water fluctuations are highly dependent on the time constant regulating surface infiltration. Flowers & Clarke (2002b) applied the model to Trapridge Glacier and successfully replicated features of the spring and autumn transitions. Although the Flowers model yields a more realistic picture of ice cap drainage than Shreve's (1972) hydraulic potential method, it has important limitations, most notably that englacial storage is "tuned" to create recognisable subglacial signals (Marshall, 2005).

Arnold *et al.* (1998) simulated the full hydrological network of Haut Glacier d'Arolla using a distributed, physically-based model. Rather than parameterising basal drainage as a two-layer distributed system like Flowers & Clarke (2002a), they specified a drainage network whose character evolved through time in response to changing meltwater inputs. Arnold *et al.*'s (1998) model is made up of three main components: (i) a surface energy balance submodel; (ii) a surface flow routing submodel; (iii) a subglacial hydrology submodel. The subglacial submodel calculates water flow along the drainage network by representing it as a series of pipes susceptible to closure from ice overburden pressure and melting from viscous heat dissipation. The location of pipes is inferred from calculations of the hydraulic potential gradient. The model's drainage configuration from distributed to channelised drainage is adjusted as the snowline passes each moulin. The model captured essential features of measured diurnal and seasonal proglacial stream hydrographs, with the best matches being obtained in the middle of the season, once the channelised system was established under most of the glacier. Willis *et al.* (2002) developed the surface melt and surface routing components of Arnold *et al.'s* (1998) model and found that the up-glacier retreat of the snowline was significant in developing a more hydraulically efficient subglacial drainage system.

1.2 Subglacial drainage in Greenland

Subglacial drainage has been investigated beneath valley glaciers (e.g. Hock & Hooke, 1993; Arnold *et al.,* 1998; Bingham *et al.,* 2003, 2005, 2006) and intermediate-sized ice caps (e.g. Björnsson, 1982; Flowers *et al.,* 2003), but there has been relatively little focus on GrIS. It is thought that warm-based conditions prevail beneath the ice sheet because thicker ice insulates the bed from the atmosphere and promotes internal deformational heating (Marshall, 2005), but the configuration of the basal drainage system remains largely unknown.

Whilst there is strong evidence that Shreve's (1972) formulation of subglacial water flow influences hydrologic routing beneath polythermal glaciers, surface meltwater accesses the bed in fewer places on an ice sheet and may therefore be less predictable in parts of Greenland. Following Björnsson (1982), most GrIS hydrology models formulate englacial drainage so that all meltwater reaches the bed of GrIS, which is assumed to be impermeable. This is a reasonable assumption for large-scale flow in regions with basal ice at PMP (Ahlström *et al.,* 2002; Colgan *et al.,* 2011a).

It is generally supposed that the drainage system near the GrIS margin is channelised (e.g. Pimentel & Flowers, 2010), mainly due to the higher slopes found in marginal zones. However, this may not be applicable to the inner ice sheet, where surface slopes are much lower (Ahlström *et al.,* 2005). Joughin *et al.* (2008a) proposed that INSAR data of seasonal speedup in western Greenland could provide a clue as to the type of drainage system found beneath the ice. Their observed speedup was spatially uniform when averaged over 24 days, which is consistent with motion over a well-distributed drainage system rather than a sparse network of large tunnels.

Lewis & Smith (2009) modelled a simple hydrologic drainage network for the entire GrIS using surface and bed DEMs, which they defined as a first-order model best suited for broad-scale hydrological assessment (Figure 1.4). Hydrologic flow networks were calculated using GrIS hydraulic potential fields and intersected with climate model simulations of surface meltwater production to identify 293 hydrologic basins. On average, ~36% of the modelled drainage network was activated (i.e. received water) over the 1991–2000 study period, with greater hydrologic activity in western regions than in eastern regions for a given latitude. Lewis & Smith (2009) hypothesised that the remaining areas, barring dynamic changes to ice-surface topography, would activate if surface melt penetrates deeper into the ice sheet interior.

1.2.1 Links between the supraglacial and subglacial hydrology on GrIS

Recent studies (e.g. Zwally *et al.*, 2002; Alley *et al.,* 2005; McMillan *et al.,* 2007; van der Veen *et al.,* 2007; Das *et al.,* 2008; Shepherd *et al.,* 2009; Bartholomew *et al.,* 2010) have highlighted an important link between supraglacial and subglacial hydrology across GrIS. Zwally *et al.* (2002) radically altered the prevailing view that surface meltwater cannot penetrate the thick, cold ice on GrIS when they observed near-coincidence of ice acceleration with increased surface melting at Swiss Camp, western Greenland. The acceleration was probably caused by increased water pressure at the bedrock interface, which is a well-known mechanism for velocity changes in alpine glaciers (e.g. Iken *et al.,* 1983). Such coupling between supraglacial and subglacial reservoirs is potentially important as it could trigger a dynamic feedback increasing the ice sheet's sensitivity to a warming climate (Parizek & Alley, 2004).

Supraglacial lakes typically cover <1% of the ablation zone in Greenland and are concentrated on the western margin, with the majority south of 70^0N (Lüthje *et al.,* 2006). Supraglacial lakes or ponds often form in the same location year-to-year (Thomsen *et al.,* 1988; Sneed & Hamilton, 2007) because of the strong influence of basal topography on ice sheet surface topography (Box & Ski, 2007; Lampkin & Vanderberg, 2011; Liang *et al.,* 2012). Sundal *et al.* (2009) identified a highly negative correlation between surface slope and the likelihood of lake formation, as steeper (and thus faster) ice leads to crevassing and surface water loss without ponding. Supraglacial lakes on the western Greenland margin typically range from a few hundred metres to >2km in diameter, but generally have shallow mean water depths of 2-5m (Lüthje *et al.,* 2006; McMillan *et al.,* 2007; Lampkin, 2011). Lake volumes are mostly in the range of 1-100x10^6 m^3 (Box & Ski (2007).

Supraglacial lakes in western Greenland evolve throughout the melt season, with volume/lake area/depth peaking at the end of June (McMillan *et al.,* 2007). Lake drainage can be extremely sudden and rapid however, with lakes draining in a matter of days (Box & Ski, 2007) or even hours (Das *et al.,* 2008). Das *et al.* (2008) documented the evolution of two supraglacial lakes >2km diameter, one of which drained through thick (980m), cold ice in two hours at an average flux of 8,700m^3s^{-1}. They reported a short-lived (<24 hours) motion response during lake drainage, consisting of uplift and surface velocity increase. An efficient, channelised network likely developed at the bed, rapidly dispersing the meltwater subglacially. It had previously been thought that large overburden pressures seriously restrict rapid channel formation on GrIS, so that a surface velocity response to meltwater penetration at the bed would be prolonged (cf. Bingham *et al.,* 2005). Shepherd *et al.* (2009) observed a seasonal speedup of 55-60% greater than the winter average once lakes started draining in the melt season, and a strong diurnal link between ice velocity and ablation. Shepherd *et al.* (2009) also found that the diurnal velocity change was independent of episodic lake drainage, implying that once drained, the surface-bed connection can remain open for the remainder of the melt season.

Three main mechanisms have been proposed to explain the dynamic response to rapid lake drainage. The first is hydraulic backpressure, which involves subglacial water being trapped near a frozen margin and thus building pressure (Andreasen, 1985). The second is longitudinal coupling, whereby outlet glaciers are highly sensitive to terminus perturbations (Price *et al.,* 2008; Nick *et al.,* 2009). The third and most important mechanism is hydrofracture, which involves the propagation of a water-filled crevasse due to the stresses at its tip overcoming the deviatoric stresses in the ice (Clarke, 2005). Crevasses can penetrate to the bed of an ice sheet if the crack remains water-filled throughout (Alley *et al.,* 2005; van der Veen, 2007), which implies the need for a large water reservoir such as a lake. Since lakes often form in the same locations year after year (Sneed & Hamilton, 2007) and can warm ice, supply water, and increase the pressure driving water flow, they probably play an important role in crack "nucleation" mechanisms (Boon & Sharp, 2003; Alley *et al.,* 2005; Bamber *et al.,* 2007). Box & Ski showed that lake outburst volumes up to 31.5x10^6 m^3 are capable of providing sufficient water via moulins to hydraulically pressurise the subglacial environment over a substantial period of time. Clason *et al.* (2012) presented a spatially distributed model for predicting the location and timing of the delivery of supraglacial melt to the ice-bed interface through moulins for Devon Island, Canada. Their sensitivity tests confirmed that ncreased surface melt production has the potential to significantly influence the spatio-temporal transfer of meltwater through surface-to-bed connections in a warmer climate (cf. Liang *et al.,* 2012).

1.2.2 The relationship between subglacial hydrology and basal motion on GrIS

Several studies (Bartholomaus *et al.,* 2008; Bartholomew *et al.,* 2011) suggest that GrIS possesses a similar basal sliding mechanism to temperate glaciers. There is much empirical evidence from temperate glaciers for an inverse power-law relationship between basal motion and effective pressure (Bindschadler, 1983), whereby variations in basal sliding velocity are due to combined changes in the rate of glacier water storage ($\delta S/\delta t$) (Bartholomaus *et al.,* 2008, 2010) and changes in the *k* factor (Iken *et al.,* 1983). Enhanced basal sliding is maintained as long as meltwater input exceeds subglacial transmissivity (i.e. $\delta S/\delta t > 0$), and is terminated once subglacial transmissivity exceeds meltwater input (i.e. $\delta S/\delta t < 0$). This explains why "bursts" of basal motion are associated with meltwater "pulses" (Schoof, 2010), while sustained meltwater input, which eventually leads to a negative rate of change of glacier water storage, does not lead to basal sliding. Colgan *et al.* (2011a) applied Bartholomaus *et al.*'s (2008) conceptual model of basal motion to the Sermeq Avannarleq flowline (western Greenland) using a 1D (depth-integrated) hydrology model. Observed periods of enhanced basal sliding from remote sensing and *in situ* velocity data corresponded to modelled periods of positive $\delta S/\delta t$, whilst periods of reduced basal sliding corresponded to negative $\delta S/\delta t$. Colgan *et al.* (2011a) thus proposed that the last 50km of the flowline experiences a basal sliding regime similar to that of a temperate glacier.

In contrast to the sliding mechanisms described above, there is increasing theoretical (Karatay, 2010; Schoof, 2010) and observational (van de Wal *et al.,* 2008; Hoffman *et al.,* 2011; Sundal *et al.,* 2011) evidence that the seasonal evolution of subglacial drainage mitigates, or counteracts, the ability of surface runoff to increase basal sliding. This indicates that the positive feedback inferred by Zwally *et al.* (2002) and others is not universally operational. A more efficient subglacial drainage system can accommodate large discharges in channels operating at lower steady-state water pressures, thereby reducing the basal lubrication effect (Bartholomew *et al.,* 2011). Schoof's (2010) theoretical model suggest that channelization and glacier deceleration occur above a critical rate of water flow, implying that higher rates of steady water supply can suppress rather than enhance dynamic thinning. This suggests that ice acceleration is driven by strong diurnal melt cycles and an increase in rain and surface lake drainage events (cf. Bartholomaus *et al.,* 2008; Sole *et al.,* 2011), rather than an increase in mean melt supply. Hoffman *et al.'s* (2011) GPS observations of ice velocity and uplift in western Greenland's upper ablation zone confirm that increased summer melting may not guarantee faster ice flow. Karatay (2011) developed a subglacial hydrology model (HYDRO) that tracks subglacial water pressures and the evolution of efficient drainage networks. By coupling HYDRO with the existing Glimmer model, Karatay (2011) showed that frictional heat flux (a function of effective pressure) caps potential runaway feedback mechanisms. This mutes the dynamic speedup response of outlet glaciers to the seasonal surface signal.

1.3 GrIS in a changing climate

1.3.1 Mass balance

Climate-model projections in the latest IPCC AR4 Report suggest that temperature and precipitation will increase over GrIS throughout the 21st century (see Figure 1.5; Meehl *et al.* 2007; Graversen *et al.,* 2011). Since melt temperatures in summer are reached over large parts of the GrIS (especially along its margin), the mass loss associated with rising temperatures is expected to more than offset the mass gain due to precipitation increases (Gregory & Huybrechts, 2006). Indeed, a doubling of ice mass loss since the late 1990s has already been observed by Rignot & Karagaratnam (2006). Hanna *et al.* (2005) found that the overall mass balance in Greenland declined from 22(±51) km^3yr^{-1} in 1961-1990 to -36(±59) km^3yr^{-1} in 1998-2003, implying a significant and accelerating recent contribution from GrIS to global SLR. This is confirmed by Cazenave *et al.*'s (2009) finding that Greenland experienced mass loss that increased its SLR contribution from 4% in 1961-1992 to 16% in 2003-2008. Graversen *et al.* (2011) estimate that GrIS will contribute up to 0.17m to SLR by 2100.

Both enhanced melting and calving at the outlet-glacier fronts are contributing to the observed mass imbalance (Krabill *et al.,* 2004; Joughin *et al.,* 2008a; Bartholomew *et al.,* 2010; Sole *et al.,* 2011). Table 1.1 provides an estimate for the different mass exchanges averaged over the entire GrIS surface. Snowfall and melt dominate the surface balance (Cuffey &

Paterson, 2010). Table 1.2 summarises recent estimates of Greenland's surface balance and the estimated losses by calving and marine basal melt.

1.3.2 Ice sheet dynamics

The effect of a warming climate on ice sheet dynamics is uncertain. A lake-tracking algorithm developed by Liang *et al.* (2012) revealed that during more intense melt years, supraglacial lakes in western Greenland drain more frequently and earlier in the melt season. In addition, lakes extend to higher elevations during warmer years, meaning that drainage events will probably occur more frequently over a larger area of GrIS in a warmer climate. Such changes in lake extent and distribution are expected to lead to an increase in the spatial and temporal frequency of surface-to-bed hydrologic connections. It is unclear if this will ultimately increase (e.g. Zwally *et al.,* 2002; Shepherd *et al.,* 2009) or decrease (e.g. Bartholomew *et al.,* 2010; Sundal *et al.,* 2011) the basal sliding sensitivity of interior regions of GrIS. Accurately predicting the future dynamics of GrIS in response to changes in surface melt will thus require better knowledge of both melt generation and the state of the subglacial hydrological system at high spatio-temporal resolution (Hoffman *et al.,* 2011).

1.4 Aims and structure of the study

1.4.1 Aims and objectives

This study seeks to investigate subglacial drainage in Greenland, its links to the supraglacial system, and its potential response to predicted climate change. Current and future surface meltwater production and routing are simulated using a model that provides the inputs to a distributed, physically based model of subglacial drainage applied to the Paakitsoq region of western Greenland. The objectives of this study are:

(i) Assess the performance of the subglacial drainage model in its present form. A variety of sensitivity tests will be employed to optimise the model set-up and to derive the appropriate *k* value for the Paakitsoq region. Results from the full-system configuration for the 2005 melt season will be compared to measured discharge at a proglacial lake.

(ii) Investigate the changes in the subglacial system across a melt season. Water pressure and conduit cross-sectional area curves will be analysed to track the spatial and temporal evolution of drainage throughout the 2005 summer.

(iii) Examine the subglacial system's response to 21st century climate warming. The drainage model will be forced using GCM data under different IPCC scenarios, and patterns of melt, discharge and water pressure for three key years will be used to illustrate the hydrological changes expected at Paakitsoq by 2100.

1.4.2 Structure of the study

In sections 3.1 and 3.2, the likely layout and structure of the supraglacial and subglacial drainage systems at Paakitsoq are examined by generating grids of hydraulic potential at the bed from a 100m-resolution DEM. In section 3.3, patterns of surface melt, refreezing and net runoff are modelled for the 2005 melt season using both a surface mass balance and a positive degree-day (PDD) approach. Sensitivity tests are performed on a simple configuration of the subglacial drainage model in section 3.4, in order to explore the effects of initial conduit size and roughness coefficient on conduit discharge, pressure and cross-sectional area. A simplified model of the entire subglacial drainage basin feeding a proglacial lake is then constructed in section 3.5, using two different assumptions about the *k* value. Modelled melt input is delivered to the bed via moulins, from where it is transferred to the margin via conduits. Total system outflow is compared to measured discharge to test the model's ability to reproduce subglacial routing at Paakitsoq. Finally, section 3.6 investigates the potential response of the drainage system to future climate change. The PDD melt model is forced using surface meteorological data for 1995-2004, and for the future using bias-corrected output from the Meteorological Research Institute CGCM-3 model forced with three of the IPCC's Representative Concentration Pathways (RCPs). Analysis is focused on three key years (2025, 2050 and 2095). The implications for future ice dynamics in the Paakitsoq region are considered.

2. Data and Methods

2.1 Study area

This study focuses on the Paakitsup Akuliarusersua basin on the western margin of GrIS, an area commonly referred to as Paakitsoq (Figure 2.1). Paakitsoq has been the focus of much scientific work because of plans for hydropower development (Ahlström *et al.,* 2007) and since the establishment of the Swiss Camp research station (~70°N, ~49°W; Figure 2.2). To the south of Paakitsoq, Jakobshavn Isbrae, Greenland's largest outlet glacier that drains 6% of the ice sheet, has experienced a dramatic phase of retreat and thinning since 1998 (Joughin *et al.,* 2008b).

2.1.1 Ice sheet mass balance at Paakitsoq

The accumulation zone at Paakitsoq extends from the equilibrium line altitude (ELA) to the summit of the ice sheet, and the ablation zone is relatively narrow and steeo (Figure 2.3; Weidick & Bennike, 2007). Winter snow cover on Paakitsoq ice <500m a.s.l is patchy and confined mainly to drifts in gullies and crevasses (Thomsen, 1988). Above the ELA, snow cover is continuous. Temperature readings taken at depth by Thomsen (1988) reveal negative temperatures in the whole ice body, with a minimum temperature of -2.1°C and a maximum of -0.6°C. Temperatures remain at -0.3 to -0.1°C between the base of the ice sheet and its uppermost 50m (Thomsen & Olesen, 1990).

Wang *et al.* (2002) used an anisotropic-ice model to show that the region is primarily in steady state, and has been close to its present form for at least 12kyr. However, ice flow around Paakitsoq displays significant seasonal velocity variability, which may be a result of local hydraulically induced basal sliding (Zwally *et al.,* 2002) and possibly the retreat of Jakobshavn Isbrae (Krabill *et al.,* 2004).

2.1.2 Supraglacial topography and drainage at Paakitsoq

Water at Paakitsoq drains from the ice sheet supraglacially or subglacially into three proglacial lakes, two of which (lakes 233 and 326) feed into the principal lake 187 (Thomsen *et al,* 1988; Figure 2.2). Numerous smaller supraglacial lakes (ranging from a few hundred metres to ~1.5km in diameter) are connected to supraglacial channels. Surface drainage patterns are determined by ice surface topography (Figure 2.4) and, to a lesser degree, by structural features such as healed crevasses and lineaments (Thomsen *et al.,* 1988).

Surface lakes drain primarily through crevasses and moulins (Thomsen & Braithwaite, 1987). Moulins vary in size depending on age and amount of water draining into them, but diameters of 1-2m are normal for the area (Thomsen *et al.,* 1989). Moulins occur with an aerial density of ~0.2km^{-2} in the region (Zwally *et al.,* 2002), and Thomsen *et al.* (1988) identified 249 separate drainage cells where supraglacial runoff flows into a moulin. The supraglacial drainage pattern takes place in the same broad valley systems from year to year (Thomsen, 1986, in Thomsen *et al.,* 1988). A recent study by Colgan *et al.* (2011b) found that the distribution of crevasses >2m wide significantly increased (13±4%) between 1985 and 2009.

2.1.3 Subglacial topography and drainage at Paakitsoq

The subglacial topography of Paakitsoq (Figure 2.5) varies in altitude from ~300m b.s.l to ~650m a.s.l. An extensive plateau-like feature reaches elevations of ~500m in the north of the area. A large NE-SW trough, which is covered in thick ice, implies that the areas around Swiss Camp may be experiencing substantial tensile stresses that could significantly increase deep crevasse formation (Price *et al.,* 2008; Colgan *et al.,* 2011b).

The natural setting of the Paakitsoq region makes dye-tracer experiments difficult (Thomsen *et al.,* 1989), so the exact structure of the drainage system and its links to proglacial outlets are not precisely known. Reeh (1983) proposed the existence of a zone 18-292km from the ice margin where basal ice is at PMP (confirmed by Wang *et al.,* 2002), and another zone 0-18km from the ice margin with fully developed bottom sliding and high water pressures. Hot-water drilling experiments carried out by Thomsen & Olesen (1990) indicate that the englacial and subglacial drainage systems in the region are well connected to the ice surface. Diurnal oscillations in subglacial water pressure are delayed by 2-5 hours relative to temperature maxima (Thomsen *et al*., 1991), suggesting that supraglacial melt flows relatively rapidly through moulins to reach the subglacial system.

2.2 Input data

2.2.1 Digital Elevation Model

The 100m surface DEM was derived from 30m ASTER GDEM data and smoothed using a 6x6 cell medium filter to remove small-scale noise, then resampled to 100m using bilinear interpolation. The bed DEM was flightline data interpolated to 750m by J. Plummer/K. van der Veen and resampled to 100m resolution by A. Banwell [University of Cambridge]. DEMs are in Polar Stereographic projection (71^{0}N, 39^{0}W, WGS84). Figures 2.4 and 2.5 display the bed and surface DEMs used in this study.

2.2.2 Proglacial discharge

Discharge was measured from lake 187 during the 2005 summer melt season at the ASIAQ 437 station (69^{0}27'58"N, 50^{0}16'53"W; 190m a.s.l.; *unpublished data*) (Figure 2.6). Discharge at lake 187 represents the cumulative discharge from lakes 233 and 326 (Thomsen *et al.*, 1988). Measurements were collected every three hours. The station at this lake measures water stage; discharge was calculated using an empirical stage-discharge relationship for the lake outflow.

2.2.3 Surface air temperature

Hourly surface air temperature data for the surface energy balance and positive degree-day melt models for 2005 are from the JAR 1 GC-NET station (69^{0}29'42"N, 49^{0}42'14"W, 962m a.s.l; Steffen & Box, 2001; Figure 2.6) Temperature data for the "baseline year" (1985-2004) are from the ASIAQ 437 station (*unpublished data*).

2.2.4 Precipitation

Precipitation data for summer 2005 and the "baseline year" of 1985-2004 were obtained from the ASIAQ 437 station (*unpublished data*). Rain gauges were designed to empty automatically when full (Petersen, 2008 *as referenced in* Long, 2008).

2.2.5 Future climate forcing

Data from the Meteorological Research Institute's CGCM-3 model, which was run (version 20110831; ensemble r1i1p1; PCDMI) as part of the 5th Climate Model Intercomparison Project (CMIP5), are used to investigate the impact of climate change on the Paakitsoq region. Monthly precipitation and temperature values from 2006-2100 were retrieved for the grid cell incorporating Paakitsoq (68°-70°N, 309°-311°E) to be used as input to this study's positive-degree-day model. The CGCM was forced with three of the IPCC's Representative Concentration Pathways (RCPs): 2.6, 4.5 and 8.5 (Figure 2.7). The RCPs are defined by their total radiative forcing (cumulative measure of human emissions of GHGs from all sources expressed in Wm^{-2}) pathway and level by 2100 (van Vuuren *et al.*, 2011). Table 2.1 describes the assumptions behind the RCPs, which are based on an internally consistent set of socioeconomic assumptions.

Meteorological output from climate models is not directly applicable for impact studies because climate models are unable to represent local subgrid-scale features and dynamics (Giorgi *et al.*, 2001), which leads to biases in temperature and precipitation and can strongly affect mass balance calculations (Radic & Hock, 2006). A simple statistical downscaling method referred to as "local scaling" (Salathé, 2005) is therefore applied to the CGCM-3 output, with measured data from the ASIAQ station used as a reference climate. The monthly climate model output series was corrected using the averaged difference over a baseline period of 20 years (1985-2005) between climate model and measured data for each month. The future temperature time series (T_i) is:

$$T_i(t) = T_{i,\,GCMf}(t) + (T_{i,\,measured} - T_{i,\,GCMh}) \qquad i = 1, \ldots, 12$$

(Equation 2)

Where $T_{i,GCMf}$ is the mean monthly temperature for the *i*th month from the future run of the GCM for t=2006-2100; $T_{i,\,measured}$ is the mean measured temperature, for the *i*th month, over the period 1985-2004; $T_{i,\,GCMh}$ is the mean temperature of historical run of GCM, for the *i*th month, over the period 1985-2004.

For precipitation, the local scaling method simply multiplies the large-scale simulated precipitation at each local grid point by a seasonal scale factor; precipitation is scaled equally throughout the year. The future precipitation time series (P_i) is:

$$P_i(t) = P_{i,\,GCMf}(t) . (P_{i,\,measured} / P_{i,\,GCMh}) \qquad i = 1, \ldots, 12$$

(Equation 3)

Where $P_{i,\,GCMf}$ is the monthly precipitation sum for the *i*th month from the future run of the GCM for t=2006-2100; $T_{measured}$ is the mean measured precipitation, for the *i*th month, over the period 1985-2004; $P_{i,\,GCMh}$ is the mean precipitation of historical run of GCM, for the *i*th month, over the period 1985-2004.

2.2.6 Satellite imagery

LANDSAT 7 Enhanced Thematic Mapper + (ETM+) images (30m pixel resolution) were downloaded from the USGS Global Visualisation Viewer (GloVis) [*https://lpdaac.usgs.gov/get_data/glovis*]. Images were acquired from 16th June and 4th September of 2005, in order to perform visual comparative analysis of the start and end of the melt season.

2.3 Modelling analysis

2.3.1 Water flow routing

Various algorithms have been developed to calculate flow accumulation in hydrological studies, but there is no single solution (Arnold, 2010). Sparse DEM data previously justified this approach, but improved resolution of DEMs from contemporary remote sensing techniques means that it is now difficult to support (Lindsay & Creed, 2006).

Arnold (2010) developed a new algorithm, which preserves catchment-scale flow connectivity without modifying the original DEM. It assumes depressions are real features in the landscape and allows them to fill and overflow into downstream areas through a continuous calculated drainage network topology, thus enabling flow continuity. “Upstream” catchments that flow into a pit or depression simply feed area to “downstream” catchments over their outlet, through a catchment hierarchy that eventually reaches an overall outlet from the DEM domain. Arnold (2010) provides a detailed explanation of the algorithm.

Routing takes place over the surface DEM, and also over the hydraulic potential surface under different *k* values. Using ice surface and bed elevation data as inputs, Shreve’s (1972) equation was employed to calculate the hydraulic pressure potential field (Equation 1). Arnold’s (2010) flow routing was then applied to create a modelled hydrological network (Figure 2.8). A ρ_i value of 917kgm^{-3} was used to match Banwell *et al.’s* (2010) model of the Paakitsoq region.

2.3.2 Supraglacial melt models

It is necessary to characterise the way water is produced and routed supraglacially, and then delivered to the subglacial environment. Two supraglacial melt models are used in this study: (i) a distributed, physically based, surface energy balance model that is coupled to a water routing and lake-filling model, and (ii) a simplified positive degree-day (PDD) model.

2.3.2.1 Surface mass balance model

Banwell *et al.* (in review:b) developed a combined modelling approach to simulate supraglacial water flow and lake filling. The SMB model consists of three coupled components: (i) an energy balance component that calculates the energy exchange between the glacier surface and the atmosphere; (ii) an accumulation routine; and (iii) a subsurface component, simulating refreezing and net runoff. The subsurface model is described fully by Rye *et al.* (2010).

Whilst the rate of water production can be calculated using the SMB model, the location and size of lakes, their catchment areas, and water routing within and between catchments are modelled by a surface routing and lake-filling (SRLF) model consisting of two main components (Banwell *et al,* in review:b). The first component uses the surface DEM to identify topographic lows containing lakes, their catchment areas, and the topological routing of water between overflowing catchments using Arnold's (2010) algorithm. The second component calculates the time delay between the production of melt and its arrival in a lake by calculating the route taken by water within each catchment and its velocity, in order to calculate input hydrographs for each lake (Banwell *et al.,* in review:b).

The energy balance model was parameterised by Banwell *et al.* (in review:a) for the 100m ASTER GDEM data. The same initial parameterisations are used in the current study, and are shown in Table 2.2. Tests by Banwell *et al.* (in review:b) indicate that the model is relatively insensitive to the hydraulic radius (R) and Manning's roughness (*n*), so constant values of R=0.035 m and n=0.05 $m^{1/3}s^{-1}$ were used in the full model run (c.f. Arnold *et al.*, 1998; Willis *et al.*, 2002).

2.3.2.2 Positive degree-day model

A simple positive degree-day (PDD) melt model (e.g. Reeh, 1989; Radic & Hock, 2011; Clason *et al.,* 2012) was used alongside the SMB model to quantify total ice surface melt using daily-averaged temperature measurements. The model is an adaptation of an original by C. Rye (unpublished). The total melt volume produced in each surface grid cell during each time interval is calculated following Arendt *et al.* (2009):

$$M = -T(z)\delta\,[T(z)]\mathrm{DDF}_{\mathrm{snow/ice}}\,\Delta t + P(z)\delta[-T(z)] \qquad \text{(Equation 4a)}$$

$$T(z) = T_{aws} + (\mathrm{z} - \mathrm{z}_{\mathrm{aws}})\Gamma_T \qquad \text{(Equation 4b)}$$

$$P\,(z) = \mathrm{P}_{\mathrm{aws}}\mathrm{k} + (\mathrm{z} - \mathrm{z}_{\mathrm{aws}})\Gamma_P\,\mathrm{P}_{\mathrm{aws}} \qquad \text{(Equation 4c)}$$

Where M is the melt water depth produced during the time interval (in mm), T is the daily average air temperature (◦C), P is the daily total precipitation (rain and snow, mm w.e.), $DDF_{snow/ice}$ is the degree-day factor for snow/ice (mm◦C^{-1}d^{-1}), z is elevation (m), Δt is the time interval 1 hour, and the subscript '*aws*' refers to values measured at the automatic weather station at JAR1. Values of T_{aws} and P_{aws} are adjusted for elevation using constant temperature and precipitation lapse rates Γ_T (◦Cm^{-1}), Γ_P (%m^{-1}). δ determines the threshold between positive temperatures for melt and negative temperatures for accumulation of solid precipitation:

$$\delta[T\,] = 1,\ T > 0; \quad 0,\ T \leq 0 \qquad \text{(Equation 4d)}$$

The simple melt model calculates spatial patterns in surface melt. Degree-day factors are calculated for a single point on the ice sheet surface, and melt is then distributed over the study area using a DEM. Melt is calculated cumulatively within the model for each day, allowing a DDF for snow to be applied initially, and a DDF for ice to be applied when cumulative melt exceeds the initial prescribed spring snowpack depth, plus any precipitation falling as snow (cf. Clason *et al.*, 2012). The model does not account for routing across the surface. The SMB model was run for one mass balance year (2003-2004), and its final snow distribution for JD 243 (31st August) 2004 was used as the initial input for the PDD model.

The concept of PDD modelling involves simplifying complex processes more accurately described by surface energy balance modelling. DDFs display significant small-scale spatial variability due to local topographic effects (Hock, 1999), as well as seasonal variability due to changes in direct radiation. Although PDD modelling can only account for the effect of temperature change on the mass balance and not the spatio-temporal variability induced by other variables (Bougamont *et al.,* 2005), PDDs are highly correlated with melt in

Greenland (e.g. r=0.96; Braithwaite & Olesen, 1989) and can provide estimates of melt that are comparable to more complex energy balance modelling (van de Wal, 1996).

The same initial parameterisations as Banwell *et al.'s* (in review:b) energy balance model were applied to this PDD model (see Table 2.2). DDF=8.9mm per PDD is used for ice in this study, as this rate is consistent with the ~8mm per PDD value measured by Braithwaite & Olesen (1989) and Braithwaite (1995) for western Greenland. DDF=3.6mm per PDD for snow in this study, following Braithwaite (1996) and McMillan *et al.* (2007). This is based on Braithwaite's (1995) assumption that the DDF for snow is 40% that of ice.

Whilst refreezing in the snowpack is of small importance at the GrIS margin, it can considerably lower the net amount of melt higher up the ice sheet (Lefebre *et al.,* 2001). Refreezing in the snow/firn pack at the start of the melting season delays water transport to the base and affects the timing of peak discharge events (van Pelt *et al.,* 2012). Given that parts of Paakitsoq are located in the accumulation zone (Weidick & Bennike, 2007), it is essential to include a formulation of the refreezing process in the region. Following Radic & Hock (2011), annual refreezing R (cm) is related to annual mean air temperature T_a (°C) by:

$$R = -0.69\ T_a + 0.0096$$

(Equation 5)

Where the lower boundary of R is 0 across the whole glacier, while an upper boundary is applied in the ablation zone and is assumed equal to accumulated snow. Monthly melt refreezes until the accumulated melt in one balance year exceeds the potential refreezing, at which point it is treated by the model as "melt".

2.3.3 Distributed, physically-based subglacial drainage model

2.3.3.1 Description of model methodology

Arnold *et al.'s* (1998) subglacial drainage model is used to route the calculated input flow beneath the ice sheet via the subglacial drainage system. This model is derived from the EXTRAN block of the US Environmental Protection Agency Storm Water Management Model (SWMM), which simulates water flow through a sewer network based on a solution of St Venant equations (Roesner *et al.,* 1988). In its form for the present study, SWMM routes flow through a series of circular conduits that join at vertical "junctions", with wider junctions (representing moulins) feeding water from specified supraglacial lakes into the system (Figure 2.9). It is assumed that most of the surface water entering moulins will flow quickly to the base of the ice sheet (cf. Björnsson, 1982; Thomsen *et al.,* 1988).

EXTRAN is considered a pseudo-2D model because conduits can branch and converge at junctions, but the model solution only requires the slope and length of a conduit as spatial parameters, and its location in series relative to other conduits. Numerical solutions for discharge in conduits and pressure in junctions are calculated using a modified Euler method with a 1-second time-step (following Banwell *et al.,* 2010) to solve the St Venant equations. Long (2008) employed a 10-second time-step, but this was found to be highly unstable for the volumes of water generated at Paakitsoq. As soon as the water level in the junctions rises above the ice sheet surface elevation, it is treated as floodwater by the model; therefore, pressure values are limited to ρ_w=1.11ρ_i, representing junction water depth equal to ice sheet thickness. Appendix A provides more detail on methodological steps of the EXTRAN block.

Arnold *et al.* (1998) made several adaptations to EXTRAN in order to adequately model subglacial drainage. They included a formulation for conduit closure and melt to calculate the net change in conduit cross-sectional area over each time-step. This is overly simplified assuming that material derivatives of temperature are negligible in comparison to other energy terms (e.g. Clarke, 2003), but it serves as a good first-order approximation of conduit melt dynamics (Spring & Hutter, 1981). The conduit closure rate is given as:

$$A = -(\rho_i - \rho_w) \left|\rho_i - \rho_w\right|^{m-1} 2(1/mB)^m S$$

(Equation 6)

Where A is the change in conduit cross-sectional area per unit time, ρ_i is ice overburden pressure (Pa), ρ_w is conduit water pressure (Pa), m is the exponent in Glen's flow law, B is the Arrhenius parameter in Glen's flow law (N m^{-2} $s^{1/m}$), and S is the conduit cross-sectional area (m^2).

The rate of conduit melting is given by:

$$M = [(\pi S)^{1/2} \rho_w (f_r v^3 / 4)] / L$$

(Equation 7)

Where M is the mass melted per unit conduit length per unit time, f_r is a friction coefficient, v is the velocity of water in the conduit (m s^{-1}), and L is the latent heat of fusion of water (kg^{-1}).

2.2.3.2 Parameter values and sensitivity tests

EXTRAN parameters are summarised in Table 2.3. The location and lengths of "moulins" (the pits in Arnold's (2010) algorithm) were calculated using data from the DEMs, and conduit length, location and slope were specified by mapping them onto the drainage network inferred from flow routing analysis (see section 3.5). Conduit diameter and roughness were specified based on physically plausible values (Arnold *et al.,* 1998) and the sensitivity tests presented in section 3.4.

3. Results

3.1 Supraglacial drainage

3.1.1 Observed features from LANDSAT imagery

Analysis of LANDSAT imagery of Paakitsoq over the 2005 summer melt season provides a useful insight into the supraglacial dynamics of the region. From mid-June (Figure 3.1a) to early September (Figure 3.1b), hydrological features on the ice sheet surface were approximately in the same location as those identified by Thomsen *et al.* (1988) using aerial photos from 1985, and by Long (2008) using LANDSAT images from 2001.

In-depth visual analysis of moulin distribution is difficult because the LANDSAT resolution exceeds the typical diameters of moulins (1-2m; Thomsen *et al.,* 1988). Nevertheless, dark spots <100m diameter are dotted around the ablation zone, and can sometimes be identified within lighter lake outlines. These spots likely correspond to moulin openings, or crevasses/deep depressions associated with moulin formation.

Comparisons between the 16th June image and the 4th September image illustrates the transience of supraglacial hydrological features. Most lakes have drained between the two images; the lake highlighted in red in Figure 3.1 is the only substantially sized lake that remains by the end of the melt season. It is approximately the same size and shape over the two images, suggesting it has probably not connected with the bed by September. There is little evidence for moulins in the September image, perhaps because moulin closure (Boon

& Sharp, 2003) occurs once lakes have drained. These observations imply there is widespread drainage into the ice sheet at Paakitsoq, as opposed to continued surface storage over the melt season.

3.1.2 Supraglacial flow accumulation

A 1km DEM of the region, interpolated from Bamber *et al.'s* (2001) dataset for the whole of Greenland, was tested in preliminary sensitivity experiments to determine whether its resolution was appropriate for calculating supraglacial and subglacial accumulation flowpaths. The 1km DEM could replicate broad drainage pathways and general bidirectional flow (see Appendix 2), but it had a smoothing effect on fine-scale dendritic networks produced by the 100m DEM. Given that ice-flow and meltwater models are highly sensitive to DEM resolution (e.g. Ahlström *et al.,* 2002; Durand *et al.,* 2011), particularly at ice sheet margins where thinner ice and complex bed topography result in hydraulic potential fields that change significantly over small distances, the 100m DEM was deemed more reliable for the purposes of this study.

3.1.2.1 Theoretical surface flow routing compared to LANDSAT imagery

Arnold's (2010) flow-routing algorithm was applied to the surface DEM, and the resulting drainage network (Figure 3.2) reflects the influence of the ice topography. Supraglacial channels feeding topographic lows correspond well with real channels and lakes, particularly the larger lakes. The accumulation of water at three main points along the ice margin corresponds well to the outflow lakes used in this study. There is a discrepancy where the flow routing algorithm predicts a long series of large supraglacial lakes in the eastern and northeast corners of the study area, without there being any corresponding lakes in the LANDSAT imagery. This may be due to insufficient melt being produced before June to generate lakes at this altitude. The correspondence between observed and modelled supraglacial drainage features breaks down as proximity to the ice margin increases. Modelled channels become more extensive the closer they are located to the ice sheet margin, whereas Thomsen *et al.*'s (1988) map shows surface drainage networks terminating in moulins rather than at the ice margin itself. In contrast, drainage at the ice margin tends to be dominated by smaller channels and moulins that are too small to be detected using a

100m-resolution DEM. Here, very few channels drain surface water directly over the edge of the ice sheet (cf. Zwally *et al.,* 2002).

3.1.2.2 Surface flow routing

The flow accumulation maps (Figure 3.3) suggest a strong bidirectional trend in surface runoff (cf.Mottram *et al.,* 2009). To the east, the influence of the Jakobshavn basin is evident, with predicted surface meltwater streams running from north to south. Otherwise, a strong east-west drainage pattern is apparent, consistent with ice flow from the ice divide westward to the margin. It appears that the "bidirectional split" has its origins relatively far inland, near Swiss Camp.

3.2 Subglacial drainage

3.2.1 Bedrock topographical controls

The bedrock topography beneath the ice sheet at Paakitsoq displays a strong directional trend, with deep northeast-southwest depressions. Many of the surface lakes in the north and central sectors tend to coincide with zones of bedrock depression below (Figure 3.4). A few of the largest lakes (labelled A, B and C in Figure 3.4) occur over deep troughs in the bedrock. Since the ice surface slopes uniformly (ignoring localised topographic lows) towards the margin, this suggests that surface ponding is concentrated over areas of greater ice thickness.

3.2.2 Subglacial flow accumulation

Changing the assumed *k* value (Shreve, 1972) can significantly alter the topology and size of subglacial drainage networks and their catchments, but it is still unclear which *k* factor should be used to characterise the subglacial system at Paakitsoq. The theoretical drainage network pathways are presented in Figure 3.5. Only *k* values of 0.50 and above have been plotted because lower values are unrealistic for most of the ice sheet at Paakitsoq. The flow accumulation maps for k=1.0 and k=0.95 produce similar dendritic networks, with a narrow, sinuous channel from northeast to southwest. As *k* decreases, this channel splits into a series of smaller subglacial lakes. A step-change in the structure of the subglacial network occurs between k=0.95 and k=0.90, as two medium lakes form in topographic depressions in the central and northern sectors. At lower *k* values (0.50 and 0.70), the flow accumulation maps show evidence of a large subglacial lake in the northeast topographic depression, feeding two or three main accumulation channels. Significant flow accumulates at the main proglacial lakes in all the maps.

3.2.3 Subglacial drainage catchments

This section investigates the effect that variations in the k value have on the size and shape of the subglacial catchments feeding lake 187. An algorithm was developed to identify all the sub-catchments that contribute to the discharge at the gauging station. Subglacial catchment shape and size remains very similar from k=1.0 to k=0.925 (Figure 3.6), with the surface area ranging from 256.6km^2 (k=1.0) to 222.0km^2 (k=0.925) (Figure 3.8). There is a significant step-change at k=0.90, signalled by the change in mean area from 247km^2 for k=1.0-0.925 to 107km^2 for k=0.90-0.50. Subglacial catchment size then remains relatively constant, ranging from 118.9km^2 for k=0.90 to 86.6km^2 for k=0.50 (Figure 3.8). The equipotential lines in Figure 3.6 are similar from k=1.0 to k=0.875, and generally correspond to the pattern of ice thickness. The equipotential lines increasingly reflect the influence of the bed as k drops below 0.80. By k=0.50, they follow large bedrock topographical depressions closely (cf. Flowers & Clarke, 1999; Rippin *et al.,* 2003). Comparing these drainage catchment delineations to those of Mottram *et al.* (2009) (Figure 3.7), we find that the catchments for k=0.90 are similar in size and shape. Mottram *et al.'s* (2009) catchment for k=0.70 is far larger, reaching further north and east.

3.3 *Supraglacial melt*

3.3.1 Measured proglacial discharge

Measured proglacial drainage at the ASIAQ 437 gauging station for the period 1st June 2005 (Julian Day 152) to 31st August 2005 (Julian Day 243) is presented in Figure 3.9. Discharge initially increases from 10m^{3}s^{-1} at the beginning of the melt season to ~60m^{3}s^{-1} in mid-June, before decreasing to 20 m^{3}s^{-1} in early July. It then increases in three distinct cyclic periods lasting 4-5 days. Discharge peaks at 129m^{3}s^{-1} on 28th July (JD 209), before decreasing in short cycles to 10m^{3}s^{-1} by late August. The original data included a significant spike in discharge on 1st August (JD 213) from ~100m^{3}s^{-1} to ~400m^{3}s^{-1}. Maximum discharge was attained within a few hours, and the spike lasted approximately three days. Discharge returned back to normal levels by the evening of 3rd August (JD 215). The likely explanation for this spike is a sudden lake outburst event (e.g. Box & Ski, 2007; Das *et al.,* 2008), but the models employed in this study are not designed to simulate such transient behaviour. Therefore, the spike was cropped from the data series (Figure 3.9b).

3.3.2 Modelling melt: SMB model

Proglacial discharge at lake 187 was simulated using the SMB model run for different *k* values. The surface drainage catchments that corresponded to the relevant *k*-derived subglacial catchments were used to "mask" the SMB model. Depending on the effect of *k* on the subglacial drainage catchment, this increases or decreases the corresponding area at the surface, which in turn affects the volume of melt predicted to reach the proglacial lake. Figure 3.10 displays the equivalent supraglacial catchments for *k*=0.95, 0.925, 0.90 and 0.875 superimposed onto their relevant subglacial catchments, and their corresponding moulin distributions. A notable step-change in the shapes and surface areas of both the supraglacial and subglacial catchments occurs between *k*=0.925 and *k*=0.90, suggesting that this interval represents a key "switching point" in the routing of water beneath the ice at Paakitsoq. It is assumed that all the melt generated at the surface rapidly reaches the bed, and flows on to lake 187; the model thus provides an upper limit for the discharge at the gauging station.

Clear diurnal patterns emerge from the melt model for all *k* values (Figure 3.11 and 3.12). These cycles track temperature changes: melt increases during the warmer periods of the day, and decreases at night. Diurnal variations in modelled melt and measured discharge generally correspond well, with diurnal peaks in measured discharge occurring a few hours after peaks in modelled melt. This suggests that surface meltwater can be routed relatively rapidly through the system. Large peaks in melt (e.g. around 1st July [~750 hours] and around 18th July [~1100 hours]) are usually followed by substantial increases in measured discharge ~5 days after the event.

24-hour moving averages of k=1.0 to k=0.925 show comparable behaviour over the three months of the study period, with melt increasing slightly from the beginning of the melt season to early July, before decreasing towards the end of August (Figure 3.12). The model runs for these *k* values tend to over-estimate the production of meltwater by tens of m^3s^{-1} at the beginning of the season, but coincide well with measured data from early July onwards. The correspondence between model runs at k=0.90 and k=0.875 and the observed data is relatively strong in the early melt season, although the model tends to underestimate melt at these *k* values in the latter half of the melt season.

Table 3.1 and Figure 3.13 present the total cumulative volumes of measured and modelled discharge over the study period. Total volume at the gauging station was $4.41 \times 10^8 m^3$. Total volume for the model runs varied from $6.08 \times 10^8 m^3$ (*k*=0.975 catchment) to $3.58 \times 10^8 m^3$

(k=0.875 catchment). Total volume decreases as the k factor decreases, although k=1.0 produces less cumulative discharge over the 92 days than k=0.975 and k=0.95. There is a significant drop in volume ($1.75 \times 10^8 m^3$) between catchments for k=0.925 and 0.90 (Figure 3.13). This supports the finding from Figure 3.10 and Figure 3.12. Melt generated towards the end of the study period may realistically not have had time to reach lake 187 by 31st August. An appropriate k value characterising the Paakitsoq area should therefore produce slightly more discharge than measured proglacial output. An "average" k value of 0.925-0.95 is probably the most suitable for the region.

3.3.3 Modelling melt: PDD model

3.3.3.1 Incorporating the refreezing process

The process of refreezing contributes significantly to the mass budget of glaciers and ice sheets (Radic & Hock, 2011; van Pelt *et al.,* 2012). Figure 3.14 shows the PDD melt model series with and without refreezing, and Figure 3.15 displays the melt volume differences between the two runs. Early in the mass balance year (~JD 250-300), spikes in melt occur roughly every week, lasting a few days and not exceeding $5 \times 10^3 m^3$ per day. The model run without refreezing produces more melt during these spikes, by an average of $600 m^3$ per spike. From ~JD 150 onwards, there is no difference between the two melt series, implying that all the grid cells have exceeded the refreezing threshold by late May.

3.3.3.2 Comparisons with the SMB model

The outputs from the PDD and SMB models are compared for the k=0.95 catchment. Table 3.2 shows the cumulative volumes derived from PDD runs with and without refreezing. Refreezing reduces the total volume by ~$1 \times 10^6 m^3$ over the study period. The modelled volumes with and without refreezing are 26.3% and 26.9% greater (respectively) than the measured volume.

The PDD melt generally tracks the SMB melt well (Figure 3.16): over the first 500 hours (~JD 152-175), the two models produce very similar series follow the shape of the measured discharge. From 500-1100 hours (JD 175-200), the PDD model underestimates melt relative to the SMB model. The SMB model predicts a significant peak in melt volume to ~$1.5 \times 10^7 m^3$, whereas the PDD model produces no such peak. From ~1500-1900 hours (JD 215-230), the modelled melt patterns track measured discharge closely, with the PDD series slightly exceeding the SMB series. In the last ~300 hours (13 days) of the study period, both models simulate oscillations in melt that are not observed in the discharge data.

Figure 3.16bis displays daily melt produced by the PDD and SMB models. The single value at JD 200 (~1200hours) that produces the peak of 1680m^3 (see Figure 3.16) is treated as an outlier and omitted. The two models show strong positive correlation, with R^2=0.699 and the Pearson's moment correlation coefficient r=0.820. The root-mean-square error (RMSE) is 782.75, and normalised RMSE is 4.67%.

3.3.3.3 Melt series analysis

"Time-slices" of the PDD model were taken every two weeks of the study period (Figure 3.17). For the first month (JD 152-194), melt appears to wax and wane: mid-level melt (30-40m^3) reaches halfway up the k=0.95 surface catchment on JD 166 and 194, whilst JD 152 and 180 experience very little melt apart from at the ice margin. On JD 208 high melt volumes of 60-70m^3 occur up to 25km inland from the ice margin. This day corresponds to peak melt at ~1300 hours in the PDD model (Figure 3.16). By JD 236, melt decreases to early summer levels, with the lower third of the surface catchment experiencing melt between 10-20m^3. As would be expected given that temperature and precipitation are a function of altitude, bands of melt (particularly distinct on high-melt days) follow ice surface elevation. The upper reaches of the k=0.95 surface catchment experience very little melt, rarely exceeding ~5m^{3}day^{-1}.

Three points were chosen along a transect (Figure 3.18), extending from the ice margin by lake 187 (point A) to the point of highest elevation in the k=0.95 catchment (point C), with point B located roughly halfway along the transect. Point A is located in the ablation zone, point B around the summer ELA (Weidick & Bennike, 2007), and point C in the accumulation zone. Melt volume per day decreases from point A to point C (Figure 3.19). Melt is produced throughout the whole study period at point A, whereas points B and C experience occasional periods of zero melt at the beginning of the summer season (JD 152), at the end of June (JD 180), and towards the end of August (JD 232). Peaks in melt volume occur simultaneously at all three sites, although volumes at point A tend to be ~3x larger than at point B, and those at point B ~1.5x larger than at point C. The 10-day moving averages show that melt volumes at all three points experience an initial rise at the end of June and a relatively long period of lower melt throughout most of July (~JD 180-200). Melt increases rapidly at the end of July to a maximum of 720m^{3}day^{-1}, 400m^{3}day^{-1} and 290m^{3}day^{-1} for points A, B and C respectively, before decreasing rapidly to zero from JD 225 onwards.

3.4 Subglacial drainage model sensitivity tests

A single, simplified channel system was constructed based on the k=0.95 catchment of the 1km DEM (see Appendix 2). This system is used to perform sensitivity tests for selected parameters of the subglacial drainage model. The single channel carries water from an "input moulin" and follows the accumulation channel for k=0.95. Figure 3.20a displays the principal subglacial water flowpath leading from the moulin to the ice sheet margin; using this map, 1000m conduits (to match the lengths used by Arnold *et al.* (1998)), are linked together along the flowpath using junctions (Figure 3.20b). The schematic representations of the conduit (Figure 3.20c) and junction (Figure 3.20d) configurations display the structure of the input system into the EXTRAN block.

3.4.1 Constant inflow to a simplified system

The following sensitivity tests employ a constant inflow (ranging from 1-10m^3s^{-1}) into the sole input junction 401 (Figure 3.20). This allows several parameters to be altered whilst controlling flow rates. A minimum conduit cross-sectional area (CSA) of 0.07m^2 was set to curtail conduit closure in regions of thicker ice. Table 3.3 summarises the initial conditions and results of the constant inflow tests. Most runs eventually display steady-state behaviour (i.e. reaching constant discharge and CSA). This contradicts Long's (2008) finding that steady-state flow occurs within a limited range of initial flow and conduit sizes. By the end of most runs, >98% of the inflow is counted as outflow.

3.4.1.1 Initial conduit size

Figure 3.21 displays the varying effects of conduit size and inflow magnitude on the size of continuity error and thus system stability. Small (1-3m) initial conduits do not suffer from closure at low constant inflows, but continuity error tends to increase as initial conduit diameter and constant inflow increase. Larger conduits can accommodate constant inflows of 5m^3s^{-1} if the junction CSA is set to 3m^2. Only run 21 could adequately cope with a constant inflow of 10m^3.

Figure 3.22 displays the discharge curves for the last conduit (24) of the system for each model run. Once water reaches conduit 24, discharge increases dramatically before oscillating slightly until reaching steady discharge. Runs with large initial conduit sizes (runs 16-23) show very little activity at conduit 24 for many days: the time taken to reach steady discharge varies from 78 hours (run 10) to 698 hours (run 23). Time to steady discharge increases with inflow rate when initial conduit diameter=1m; for all other conduit diameters, time to steady state decreases with increasing inflow.

Conduit 24's CSA curves are (Figure 3.23) show that the final CSA for each run depends largely on the inflow rate. Runs 5, 10 and 15 ($5m^3s^{-1}$) converge at $8m^2$, runs 4 and 9 ($4m^3s^{-1}$) converge at $6.44m^2$, runs 3 and 8 ($3m^3s^{-1}$) converge at $4.74m^2$ and runs 2 and 7 ($2m^3s^{-1}$) converge at $3.30m^2$. Run 1 does not reach the same CSA as run 6, possibly because its constant inflow of $1m^3s^{-1}$ does not provide sufficiently fast flow to melt the conduit walls. The time taken to reach steady CSA decreases as conduit diameter increases. Runs 16-23 are excluded from the figure as their CSAs remain at their initial size throughout the whole study period.

3.4.1.2 Manning roughness coefficient

Channel roughness can be several orders of magnitude more important than channel discharge in determining water pressure and discharge in a basal system (Röthlisberger, 1972; Ahlström *et al.* 2005), so the sensitivity of the system to Manning roughness coefficients (n) is tested, for n=0.01, 0.05 and 0.08. The configurations of runs 3 and 8 (section 3.4.1.1) are used. For run 3, increasing n significantly delays water from reaching conduit 24 (Figure 3.24). For run 8, the effect of n is not as significant, and indeed the reverse of run 3: steady discharge is reached earliest for n=0.08.

Figure 3.25 shows how n affects the CSA of conduit 24. When n=0.01, the system becomes unstable as the CSA gets larger; by the end of the study period, it has yet to stabilise (Figure 3.25a). When n=0.05 and 0.08, CSA eventually stabilises (Figure 3.25b). Like with discharge, variations in n have a greater effect on CSA evolution during run 3, which has a smaller initial conduit size.

3.4.2 Melt model input to a simplified system

An individual melt hydrograph produced by the SMB model, corresponding to the nearest pit depression, was used as input to the simple system. The modelled period includes cycles of rising and falling melt production. The system configuration is identical to the constant inflow tests. The subglacial drainage model was run twice using the melt model input: once with an initial conduit diameter of 2m ("run 1"), and once with an initial conduit diameter of 5m ("run 2"). Junction CSAs were set to $3m^2$.

Melt input exceeds modelled outflow by a factor of ~2 (peak-to-peak) throughout most of the study period (Figure 3.26); this is due to the drainage capacity of the inflow junction being exceeded early on and water overflowing onto the surface. Outflow then decreases in low-amplitude diurnal cycles. Outflow peaks and troughs occur 30-100 hours later than corresponding changes in input. Runs 1 and 2 both produce very similar outflow hydrographs.

For both runs, the CSA of all "inland" conduits (i.e. up to conduit 16) decreases from 3.14m^2 (run 1) and 19.6m^2 (run 2) to 0.2-0.3m^2 in the first 200 hours (Figure 3.27). Inland CSA curves then experience four cycles coupled to peaks in discharge (see Figure 3.26). The "marginal" conduits (i.e. conduits 23 and 24) decrease slightly in size over the first 100 hours of both runs, and then increase in low-amplitude cycles before plateauing off at 4.5m^2 (conduit 23) and 6.4m^2 (conduit 24) for run 1, and at 18m^2 (conduit 23) and 19.6m^2 (conduit 24) for run 2.

Figure 3.28 shows the pressures in selected junctions for each time step. SWMM provides the height of water in each junction as an output, and this can be taken as a measure of system pressure if calculated as a function of ice overburden. A k factor of 1.11 represents water that has overflowed from the junction onto the ice surface (since water is denser than ice). As the inflow moulin, junction 401 receives high melt input and therefore often overflows. Junction 1011 (located halfway down the subglacial system) oscillates between a k=0.75-0.05, responding to diurnal changes in melt input and to overflowing at junction 401. For both runs, peaks in pressure at junction 1011 occur ~200 hours (8 days) later than the corresponding peaks in discharge/CSA. Pressure variations in marginal junction 1023 differ between runs 1 and 2. For run 1, peaks in pressure at junction 1023 roughly coincide with peaks in discharge, whereas for run 2, the pressure remains at a constant 0.5 throughout. The difference between both runs owes to the differential size of marginal conduits; marginal conduit CSAs are larger during run 2, allowing for more water to be accommodated in the conduits and less water being stored in corresponding junctions. Junction 402 is short (Figure 3.20b) and acts as the outflow junction, so pressures remain low (k=0.05).

3.5 Subglacial drainage model for Paakitsoq

3.5.1 Model configuration

In order to model subglacial drainage at Paakitsoq, a system composed of conduits, junctions and inflow moulins was constructed based on the subglacial flow accumulation for *k*=0.95 and *k*=0.925 (Figure 3.29, Figure 3.30 and Figure 3.30bis). These *k* factors produce modelled melt volumes similar to measured melt (Table 3.1) and represent a reasonable approximation for the pressurised flow conditions that probably exist under most of the ice sheet at Paakitsoq (e.g. Thomsen & Olesen, 1991).

Pits in the surface DEM were identified as the location for input moulins (black circles in Figure 3.30bis), so that every lake has a moulin at its base. ~1000m-long conduits were used to link the input moulins together in a network topology that best matched the subglacial flow accumulation channels in Figure 3.5. Initial conduit diameter was set to 2m and the minimum conduit CSA was set to $0.2m^2$, after preliminary tests revealed conduit closure and extended periods of low flow led to high continuity error. The CSA of moulin junctions was set to $500m^2$ as this was found to optimise system continuity, whilst the remaining junctions were set to $1m^2$. Manning roughness coefficient was set to 0.05.

In order to differentiate between distributed and channelised flow beneath Paakitsoq, bundles of four parallel conduits were used to replicate distributed flow along all first-order streams (i.e. a stream that does not have any other streams feeding into it). First-order streams are mainly found beneath the inner ice sheet, where distributed drainage is likely. The bundles increase the surface area of flow, a key characteristic of distributed systems. Channelised flow is formulated by using single conduits for all second-order streams or higher.

Input hydrographs for the moulins (Figure 3.31) were generated using the melt output from SMB model, routed from each sub-catchment (insets in Figure 3.29 and Figure 3.30) to its appropriate moulin using the surface routing and lake-filling (SRLF) model. For simplicity, it is assumed that the water flowing into moulins drains through the ice instantly to reach the bed (cf. Björnsson, 1982), thus not allowing the water to accumulate as lakes that could potentially overflow. Strong diurnal melt variations are evident in the hydrographs. The maximum melt input ranges from $0.03m^3s^{-1}$ (input moulin 446) to $13.80m^3s^{-1}$ (input moulin 602).

3.5.2 *k*=0.95 catchment

Specific conduits and junctions were selected to perform discharge/pressure analysis on the subglacial model output. The conduits and junctions were chosen to provide a sample from all three main “branches” in the system as well as the main downstream channel they feed, varying in distance from the ice sheet margin (Figure 3.32).

3.5.2.1 Discharge

The total inflow volume for the main system was $3.92x10^8 m^3$, and total outflow volume at junction 702 was $3.57x10^8 m^3$. 91% of the inflow travelled through the system to its outflow point, a significant improvement on the ~10% observed by Long (2008). Total discharge from the catchment, including the marginal moulins feeding lake 187, was $5.48x10^8 m^3$. This is an overestimation of the measured discharge ($4.41x10^8 m^3$). Continuity error was -0.23%. The discharge curves for the selected conduits are shown in Figure 3.33, split into their respective branches for comparison.

System discharge shows similar behaviour across all three branches (Figure 3.33a,b,c), increasing from early June to late July, before decreasing in August; this follows the general shapes of the system's input hydrographs. Most of the selected conduits do not experience discharge $>20 m^3 s^{-1}$. Inland conduits (6, 19, 64, 80, and 112) receive very little ($<5 m^3 s^{-1}$) discharge throughout the entire study period. Once the three branches join at junction 1035, discharge increases dramatically in conduit 129, 132, 140 and 148 (Figure 3.33d).

Figure 3.34 compares the total modelled melt across the k=0.95 catchment, the total modelled inflow, modelled outflow taken at conduit 148, and the measured outflow. As meltwater does not travel instantaneously to the outflow conduit, a lag time is observed between the modelled melt, modelled inflow, and modelled outflow series. The lag time between melt production and moulin inflow is noticeable in the first half of June (0-300 hours), but shortens as the melt season progresses. By ~500 hours (JD 172), melt inflow responds very quickly (<6 hours) to melt production. Diurnal variations in flow input manifest themselves rapidly in the modelled outflow, particularly from ~500 hours onwards. For instance, a steep drop in melt input at 1300 hours is transmitted as an outflow signal within a few hours. Modelled outflow overestimates discharge by $5\text{-}70 m^3 s^{-1}$ in the first half of the study period. It tracks the measured discharge closely from 1200 hours (JD 200) onwards.

Figure 3.35 is a scatterplot relating 3-hourly measured discharge to 3-hourly modelled discharge. Relatively good correlation is observed between corresponding hourly values, with R^2=0.332 and the Pearson's moment correlation coefficient r=0.576. The RMSE is 36.10, and the normalised RMSE is 19.4%. Correlation becomes more dispersed as discharge increases.

3.5.2.2 Pressure

In branch 1 (Figure 3.36a), a distinct split in water pressure can be observed between inland junctions (481, 494, 519, 1014 and 10221) and more marginal junctions (582 and 619). Inland junctions quickly overflow and then oscillate between *k*=0.65 and *k*=1.0, whilst marginal junctions oscillate between *k*=0.4 to *k*=0.6. The selected junctions in branch 2 are located inland, so display pressure variations similar to junctions 1014 and 10221. All junctions in branches 1 and 2 show two distinctive peaks in pressure at 1250 hours and 1700 hours, corresponding to peaks in the input hydrographs (Figure 3.33).

Junctions 603 and 1063 in branch 3 (Figure 3.36c) display more stable pressures, plateauing at *k*=0.75 and *k*=0.65 respectively, whilst the more marginal junction 624 is characterised by *k*=0.4-0.5. The three junctions in the main channel also display stable pressure for the majority of the study period (*k*=0.5-0.6 for junction 656 and *k*=0.8 for junction 1081). The high pressures at junction 1081 are attributable to the large amounts of discharge flowing through it. A runaway effect of wall melting (due to thin ice at the margin) explains the low pressures at junction 1084; indeed, its neighbouring outflow junction, n°702, has a CSA of 2283m^2 at the end of the run.

3.5.2.3 Conduit cross-sectional area

Figure 3.37 displays the CSA variations in conduits 25, 51, 88 and 128. These conduits are typical junctions for the inland (n°25 and n°88), marginal (n°51 and n°128) and main channel (n°140) sub-systems identified above.

The CSA curves support the inland/marginal split in subglacial behaviour. Inland conduits 25 and 88 experience the same pattern of CSA change as marginal conduits 51 and 128, but remain 2-3x smaller throughout the study period. Four peaks in CSA (at ~800, 1200, 1400 and 1700 hours) coincide with peaks in pressure throughout the whole system (Figure 3.36a,b) and to a certain extent discharge (Figure 3.34). There is very little lag between the four inland and marginal CSA curves. CSAs decrease to 1-2.5m^2 by the end of August. Conduit 140 experiences a rapid, large change in CSA, which increases throughout the study period before plateauing to 78m^2 at ~1500 hours. The ice is only 60-120m at this conduit, which allows wall melting from high discharge to rapidly overcome creep closure.

3.5.3 *k*=0.925 catchment

The selected conduits and junctions for discharge and pressure analysis for the *k*=0.925 system are shown in Figure 3.38. Two "branches" are identified here, as opposed to the three for the *k*=0.95 system.

3.5.3.1 Discharge

Total system inflow volume was $3.24 \times 10^8 m^3$, and total outflow volume at junction 702 was $2.81 \times 10^8 m^3$. 87% of the flow travelled through the system to the outflow point. Total discharge for the catchment was $4.77 \times 10^8 m^3$, which is closer to the total measured discharge than the k=0.95 system. Continuity error was 0.95%.

Discharge across each branch was very similar to the k=0.95 catchment, so the individual series are not replicated here. Figure 3.39a displays the total modelled inflow and outflow for the k=0.925 catchment. The modelled outflow hydrographs for k=0.925 and k=0.95 have virtually the same shape (Figure 3.39b), although outflow for k=0.925 is generally lower than for k=0.95 (by up to $15 m^3 s^{-1}$). Both series tend to exceed the measured outflow, particularly from 400-1200 hours (mid-June to mid-July), but underestimate discharge during the possible lake outburst event in early August.

3-hourly measured and modelled discharge for the k=0.925 system are relatively well correlated, with R^2=0.311 and the Pearson's moment correlation coefficient r=0.557. These values are slightly lower than for the k=0.95 system. The RMSE is 32.04, and the normalised RMSE is almost exactly the same as for the k=0.95 system, at 19.3%.

3.5.3.2 Pressure and conduit cross-sectional area

The pressure and CSA curves for the k=0.925 system are very similar to the k=0.95 system, so they are not presented here. The same inland/marginal split in behaviour was simulated.

3.6 Modelling the response to future climate change

3.6.1 Model set-up and bias correction

The monthly precipitation and temperature values produced by CGCM-3 for the period 2006-2100, which was run for RCP 2.6, 4.5 and 8.5, are used as inputs to the PDD model. GCMs model "climate" and not "weather", so mean observations from 1985-2004 from the ASIAQ station are used to bias-correct the output from a historical run (1850-2005) of CGCM-3. Figure 3.40 displays the mean monthly temperature and precipitation data from the weather station and the historical run, and the "delta factors" calculated using Equations 2 and 3.

The temperature curves in Figure 3.40a show a significant difference between modelled and measured temperature, particularly in the winter months. From June to September, the two temperature curves are very similar, with delta factors dropping just below 1 for July and August. Precipitation curves also diverge, with a large modelled peak from June to September not evident in the measured data. The delta factors support Xu's (1999) observation that climate models tend to underestimate large amounts of precipitation and overestimate small amounts. The differences between measured and modelled climate variables are to be expected in this kind of study, as GCMs are not able to resolve local weather conditions (Radic, 2012, *pers. comm*).

3.6.2 PDD model forced with RCPs

3.6.2.1 Temperature and precipitation changes from 2006-2100

Figure 3.41 displays total monthly precipitation for the bias-corrected runs. Precipitation generally totals 0-150mm per month, although there are occasional peaks exceeding 200mm. Few large peaks occur in the RCP 2.6 run, whereas 7 months in the RCP 8.5 run exceed 200mm per month.

Annual precipitation trends are shown in Figure 3.42 using the 1-year and 5-year moving averages. Under all three scenarios, precipitation remains relatively stable until 2050, and even markedly decreases from 2045 under RCP 2.6. It then increases over the next 50 years in RCP 2.6 and 8.5. Under RCP 4.5, precipitation decreases over a decade from ~2075-2085, before sharply increasing over the next two decades.

Figure 3.43 displays monthly mean temperature for all three runs. Under RCP 2.6 and 4.5, most annual cycles have a range falling within -20°C to 10°C, although a few years exceed this. The 1-year (Figure 3.44a) and 5-year (Figure 3.44b) moving averages suggest that temperatures will rise until ~2050 under all three scenarios, at which point their trajectories will diverge. Under RCP 2.6, temperatures will decrease in the second half of the century to ~1°C higher in 2100 than in 2010. Under RCP 4.5, temperatures plateau, and increase slightly around 2085, before reaching temperatures ~2°C higher than in 2010. Under RCP 8.5, temperatures rise dramatically in the last quarter of the century, ending ~7°C higher than in 2010.

3.6.2.2 Forcing the PDD model

In order to force the PDD model (which requires daily weather data) with future monthly climate data, the "average" temperature/precipitation per Julian Day was calculated using a ten-year (1995-2004) baseline period of measurements to create an aggregate "baseline year". This assumption is not applicable to precipitation, however, as precipitation occurs or it does not. Therefore the mean number of "precipitation days" per month per year is used. Appendix 4 provides details on how the "aggregate" baseline temperature and precipitation series were calculated.

The PDD model was run using the output from all three RCPs to provide melt series at three key points over the next century: 2025, 2050 and 2095. A mean of the climate data from the decade around the target year (e.g. 2020-2030 for 2025) is used to improve reliability. The PDD model was run over one full mass balance year, and the period JD 152-243 was extracted for further analysis. Figure 3.45 shows the end-of-season (i.e. JD243) snow depth distribution for the mass balance year 2005; the surface catchment does not contain any snow by this time. As temperatures are predicted to increase over the next century in most CGCM-3 runs, it is expected that there will be little if any net snow accumulation relative to 2005. Furthermore, snow remaining from the previous mass balance year is likely to have an albedo closer to that accounted for by the DDF for ice. Therefore, the initial snow depth distribution for the runs was assumed to be zero across the entire surface catchment.

3.6.2.3 Temporal analysis of melt series

Figure 3.46 displays the total modelled cumulative melt volumes over the study period for the various RCPs and target years. RCP 2.6 runs show an increase in melt production by 2050 to $5.30 \times 10^8 m^3$, followed by a decrease by the end of the century to levels below that of 2005. For all scenarios, melt volumes in 2025 are very similar to that of the baseline year ($4.01 \times 10^8 m^3$). Only three of the nine runs (4.5 [2095], 8.5 [2050] and 8.5 [2095]) exceed the total volume modelled by the SMB and PDD model for the 2005 melt season. By 2050, however, all three scenarios predict total cumulative melt exceeding the measured melt for 2005 and the modelled melt for the baseline year.

The predicted melt series for 2025, 2050 and 2095 under the RCP scenarios are shown in Figure 3.47, in relation to the melt seasons of the baseline year and 2005. All three years in both RCP 2.6 and 4.5 runs have relatively similar melt series, with melt generally increasing until mid-July (~JD 190), plateauing, and decreasing sharply from mid-August (~JD 225). The 2025 melt series for RCP 8.5 tracks the baseline year closely, and by 2050 the RCP 8.5 melt series is significantly more intense than the baseline year, producing roughly double the melt per day from early June to late July (>1x10^7m^{3}d^{-1}). The 2095 melt around three to four times more intense than previous years, with the whole of July and most of August (JD180-230) experiencing melt volumes >1.5x10^7m^{3}d^{-1}.

The modelled melt for 2005 often exceeds the "average" melt for most scenarios by the end of the century, particularly at the beginning of summer. In six of the nine runs, more than 50% of the daily melt volumes are smaller than the corresponding volumes in 2005 (Figure 3.48). Only 5% of the daily volumes in run 8.5 [2095] are exceeded. The proportion of days exceeded by the 2005 series decreases later into the 21st century, but remains relatively high by 2095. This suggests that present-day inter-annual variability can result in high melt years that, under some scenarios at least, exceed "average" melt production at the end of the century.

3.6.2.4 Spatial analysis of melt series

Figures 3.49 to 3.51 display the total melt over each individual month of the melt season under the various scenarios and the baseline year. Under RCP 2.6 (Figure 3.49), melt >1500m^3 is confined to <10km of the margin during June and August, with a "hotspot" of melt in the southwest corner of the study area. Melt occurs further up the ice sheet in July, but these incursions are not much more intense nor extensive than during the baseline year. There is little variability in the spatial pattern of melt across years for respective months.

Under RCP 4.5 (Figure 3.50), more substantial melt incursions occur in July, with the same hotspot in the southwest experiencing melt >2500m^3 in July 2095. Melt in July 2025 is already more extensive and more intense than the baseline year, and by the end of the century melt >1500m^3 occurs up to 30km inland from the margin. However, melt in June and August remains similar to that of the baseline year, even by 2095.

Under RCP 8.5 (Figure 3.51), there is a noticeable change in melt extent and intensity in 2050. July melt is particularly intense, exceeding the spatial distribution of melt >2500m^3 for July 2095 under RCP 4.5. Several zones along the margin even experience melt of ~2000m^3 in August 2050. By 2095, intense (up to 3000m^3) melt occurs all along the margin in June, and extends deep into the ice sheet in July, when the majority of the k=0.95 catchment experiences melt >2000m^3. Even in August 2095, melt >1500m^3 occurs 50km inland of the margin.

Figure 3.52 displays the total cumulative monthly melt for the 2005 melt season and for the 2050 melt season under the three RCPs. June 2005 experiences more extensive melt >1000km^3 than all three scenarios, although it does not experience as many marginal "hotspots". Medium-intensity (1000-2000m^3) melt in August 2005 is not as spatially extensive as in 2050. Melt in July 2005 is similar to 2050 under RCP 2.6 and 4.5, particularly 5-20km from the margin, but future scenarios (particularly RCP 8.5) indicate there will be much more intense melt in the southwest corner than was observed in 2005.

Figure 3.53 shows "time-slices" of cumulative melt at the end of each summer month of 2095. Melt accumulates rapidly under RCP 8.5, with most of the ice sheet within 15km of the margin experiencing cumulative melt of >4000m^3 by 31st July 2095. Under RCP 4.5, patches of high-intensity melt reach further into the ice sheet, and the southwest corner experiences cumulative melt >6500m^3 over the course of the summer. Under RCP 8.5, exceptionally high melt (>7000m^3) occurs up to 15km from the margin, with the majority of the k=0.95 catchment experiencing cumulative melt >5000m^3.

3.6.3 Subglacial drainage model forced with RCPs

The subglacial drainage model was forced with the RCP melt series to investigate how the subglacial hydrology at Paakitsoq might change over the 21st century. The melt series were fed into the SLRF model to determine the distribution of meltwater to each pit depression in the Paakitsoq domain. The SLRF model requires hourly data, so the daily melt rates from the PDD model are applied hourly. This dampens actual diurnal melt fluctuations, but produces realistic daily melt volumes.

The subglacial model was configured in the same way as in section 3.5.1. The above analysis suggests that the k value varies spatially across the ice sheet as well as temporally, so it is unclear what the "correct" subglacial network configuration might be in the future. The k=0.95 system has been shown to be a reliable predictor of outflow volumes and discharges for Paakitsoq in 2005, and given that the 2005 melt series is similar to (and often exceeds) the "average" melt series of most of the RCP runs (Figure 3.47), it is considered a reliable approximation of the network structure for the subglacial drainage model.

3.6.3.1 Discharge

The total modelled outflow discharge curves for all RCP runs (Figure 3.54), broadly track the melt series in Figure 3.47, as is expected given the subglacial model configuration is consistent from run to run The maximum peak in discharge tends to occur earlier in the melt season as the century progresses, with maximum peaks occurring earlier under the RCP 8.5 scenario than RCP 2.6 and 4.5. The measured discharge in 2005 is greater than most of the modelled future series in June, but from mid-August onwards, modelled discharge significantly exceeds measured discharge.

Figure 3.55 displays the inflow and outflow discharge series for the three RCP runs in 2095, normalised to the mean of their respective inflow series. It is not possible to constrain the lag time at a finer temporal resolution due to the nature of the GCM-derived data. RCP 8.5 [2095] is more centred around its mean than RCP 2.6 [2095], suggesting that discharge in the former scenario will be higher than the latter and more consistently so over an end-of-century melt season.

Although most scenarios produce total cumulative outflow that is larger than the baseline total (Figure 3.56), the total 2005 measured discharge is only exceeded by the RCP runs in 2050 and later. The total outflow by the end of the century under RCP 8.5 could be over three times larger (12.653 $x10^8 m^3$) than the outflow in the baseline year.

3.6.3.2 Pressure

Figure 3.57 displays the pressure variations in junctions 481, 624 and 1081. These junctions are typical junctions for the inland, marginal and main channel sub-systems (respectively) identified above.

Junction 481, located inland along branch 1, displays significant variability across RCPs and across years. It initially overflows in all runs, but does so earlier in the melt season as the total discharge increases (within 150 hours under RCP 8.5 [2095], and ~800 hours under RCP 2.6 [2025]). Junction 481 does not reach steady state by the end of any melt season, although it oscillates around k=0.85 from mid-July in most cases.

Junction 624 is a marginal junction located along branch 2. Although it generally reaches k=1.11 quickly (within the first 200 hours in all cases apart from 2.6 [2025]), the pressure tends to decreases steadily throughout the study period, from k=0.6 at ~500 hours to k=0.3 at ~2000 hours. The pressure rapidly drops to k=0 in the last hours of the runs. There is less variation across runs for junction 624 than for junction 481, although pressures are reached significantly quicker as the century progresses under RCP 2.6. Under all RCPs, the pressure in junction 624 tends towards its oscillatory equilibrium slightly quicker towards the end of the century.

Junction 1081, located on the main downstream channel, varies very little across RCPs and across years. It experiences a higher peak in pressure in the first few days of the RCP 8.5 [2095] run, probably because of the large levels of total system discharge in comparison to the other runs during this period (see Figure 3.49). Junction 1081 takes slightly longer to reach steady state under RCP 2.6, which for all runs is k=0.8. This steady state is similar to that found during the 2005 melt season (Figure 3.36).

4. Discussion

4.1 Lake drainage

Satellite imagery analysis and theoretical surface flow routing maps suggest that lakes and surface streams are long-lived features at Paakitsoq. They accumulate year-to-year in the same general pit depressions in the ice surface, supporting the notion that the bed strongly influences ice sheet surface topography (Box & Ski, 2007; Lampkin & Vanderberg, 2011). The contrast between the supraglacial situation in June 2005 and September 2005 confirms the lake evolution patterns observed by McMillan *et al.* (2007) and Box & Ski (2007), whereby lakes begin to drain in late June or early July. Large outburst events probably occur at Paakitsoq, as inferred by the the significant peak in measured discharge in early August (Figure 3.9a).

4.2 Melt modelling

The PDD and SMB models show significant agreement on temporal melt production patterns over the 2005 melt season (Figure 3.16), which track the measured discharge series well (excluding the late July to early August period, when the discharge is underestimated by 40-50%). This supports van de Wal *et al.*'s (1996) finding that PDD models can produce realistic volumes of melt over an ice sheet, although this is likely to depend on the chosen DDFs for snow and ice.

Initial differences between measured discharge and modelled melt are due to unrealistically fast meltwater routing to the ice margin without storage in various elements of the ice sheet system (e.g. surface ponds, crevasses, englacial channels). As the early season snow pack melts, there is increasing correlation between measured and modelled data. Since the snow distribution from the end of the 2004 melt season was used as an initial input to the PDD model, it assumes that ice is reached as soon as the 2005 winter accumulation is lost. In reality, layers of firn from the preceding accumulation seasons still cover the ice at this point; given that firn has a higher albedo than ice (Oerlemans & Knap, 1998), actual melt will be lower than modelled melt as the model is formulated on the assumption of a "less reflective" surface that absorbs more heat.

Refreezing has been incorporated successfully into this study's PDD model. It is an important process to include for Paakitsoq because parts of the ice sheet are located in the accumulation zone (Weidick & Bennike, 2007). However, very little refreezing in the snow and firn pack occurs at the start of the 2005 melt season (Figure 3.14, 3.15). Although it is beyond the scope of this study to perform in-depth sensitivity testing of the supraglacial system at Paakitsoq to refreezing, experiments (e.g. McMillan *et al.,* 2007; van Pelt *et al.,* 2012) have shown that refreezing is sensitive to changes in parameters that affect the surface temperature, snow thickness and snow density.

Despite its strong performance over a present-day mass balance year, there are drawbacks to the PDD approach for future climate simulation. PDD models cannot account for the spatio-temporal variability induced by variables other than temperature and precipitation, which becomes limiting for future simulations as modifications to atmospheric patterns are not accounted for explicitly (Bougamont *et al.,* 2005). Furthermore, sensitivity to warmer temperatures is enhanced with a PDD compared with an energy balance model (Bougamont *et al.,* 2005), suggesting that PDD models may produce less realistic melt volumes in a warming climate.

4.3 The subglacial drainage model

4.3.1 Parameters

Arnold *et al.'s* (1998) subglacial drainage model was tested extensively. Results from the constant inflow sensitivity tests suggest that initial conduit size is important in dictating the capacity for the sub-system to accommodate all the meltwater input. If constant inflow is >5m^3s^{-1}, a conduit diameter of 3m is needed to ensure a stable system. Channel roughness has a noticeable effect on CSA and discharge when conduit diameters are 1m, but a larger initial conduit size dampens the importance of channel roughness as a controlling parameter on discharge and CSA.

Whilst a constant input of water almost always allows the conduits to reach steady discharge and CSA, inflow series from actual melt seasons present a challenge to the subglacial model. Realistic meltwater inputs contain large peaks, which can exceed the transmissivity of the subglacial system (Bartholomaus *et al.,* 2010), as well as troughs, which can lead to conduit closure if low flow is sustained over a long enough period of time. Increasing the minimum size to which a conduit can close was an important way of improving system continuity and preventing blow-up. For the relatively low discharges of the sensitivity tests, a minimum CSA of 0.07m^2 was enough to curtail conduit closure. However for the full system, whose conduits are sometimes located under very thick (up to 690m) ice, minimum CSA had to be set to 0.2m^2 to optimise continuity. This minimum CSA is both necessary in the model and physically plausible, as few conduits, especially in marginal areas, tend to shut down during the summer.

4.3.2 Inland and marginal systems

Discharge, pressure and CSA analysis of the subglacial model results provide evidence for a split in "inland" and "marginal" drainage. Figure 4.1 displays the proposed spatial split in drainage behaviour at Paakitsoq. The inland system is delimited to the ablation zone >3-10km from the ice sheet margin, based on discharge, pressure and CSA analysis for individual conduits. The corresponding supraglacial area is characterised by abundant surface channels feeding large lakes, which are connected to the bed via moulins. Inland subglacial conduits display low discharge magnitudes (<5m^3s^{-1}) during the full system runs (Figure 3.33), and high water pressures (k>0.65) that oscillate strongly according to melt input (Figure 3.36). Inland conduits display the same pattern of CSA change as marginal conduits, but remain 2-3x smaller throughout the study period. Due to high overburden pressure, inland conduits are too small early in the melt season to accommodate sudden increases in discharge. Thus limits the amount of inflow that can be evacuated from the input moulins, casusing surface flooding in the model.

The marginal system is constrained to within 3-10km of the ice sheet margin. The supraglacial drainage system is characterised by small moulins with very little surface ponding, and less well developed drainage channels. The LANDSAT imagery (Figure 3.1) provides few clues as to the existence of crevasses in the lower ablation zone, but they may be an increasingly important form of surface-to-bed connection in a warmng climate (Colgan *et al.,* 2011b). Evidence from the flow accumulation and routing algorithms suggests that few marginal catchments drain surface flow from the edge of the ice sheet (cf. Zwally *et al.,* 2002). Marginal conduits display relatively high discharge magnitudes (5-20m^3s^{-1}), with discharges in the main downstream channel experiencing discharge up to 120m^3s^{-1} in mid-July. CSAs grow with increasing proximity to the margin, and pressures in marginal junctions mostly oscillate between k=0.3 to k=0.6.

The characteristics of the inland and marginal systems are consistent with expected flow behaviour beneath ice of different thicknesses. The inland system remains active throughout the whole season despite low discharges and widespread conduit size reduction in the first days of the study period, although it is unclear whether this is a function of the model's minimum CSA setting, or if even very low (<0.1m^3s^{-1}) discharge is sufficient to keep a conduit open. The thinner marginal ice results in lower overburden pressures (Shreve, 1972) allowing conduit CSAs to grow, thus lowering water pressure. Increased discharge downstream and greater meltwater input from moulins in the lower ablation zone result in enhanced wall melting, which also initiates reduced water pressures in marginal areas (Schoof, 2010). There is little, if any, lag between the inland and marginal pressure and CSA curves for the full system (Figure 3.36 and 3.37), suggesting that peak meltwater events propagate rapidly through the system. The characteristics of Paakitsoq's marginal subglacial drainage are similar to those modelled by Arnold *et al.* (1998) for an alpine glacier, which is reasonable given the similarities in ice thicknesses.

4.3.3 The appropriate *k* value for Paakitsoq

Little is currently known about the proportion of distributed and channelised drainage at Paakitsoq, although it is generally supposed that the drainage near the GrIS margin is channelised (e.g. Pimentel & Flowers, 2010), whilst the inland drainage is most likely composed of linked cavities (Walder & Fowler, 1994). Although this study does not explicitly examine different drainage morphologies, analysis of *k* factors and the marginal/inland behaviour can provide an insight into their inferred distribution.

The evidence presented in this study suggests that an overall *k* factor between k=0.90 and 0.95 is appropriate for modelling the subglacial hydrology at Paakitsoq. The total cumulative melt volumes (Table 3.1) and subglacial drainage catchment shapes and sizes (Figure 3.6 and 3.8) for different values of k point to a step-change between k=0.925 and k=0.90. The modelled subglacial outflow using these two k values is similar to the measured discharge series, although the model runs tend to overestimate discharge in the first half of the season and underestimate it in the latter half. The k=0.95 run correlates better than k=0.925 with measured discharge, but the difference is anecdotal.

These high values of k effectively assume that drainage in the study area is distributed, but in reality the k factor varies dramatically across the ablation zone, so that an "average" k factor is an unreliable approximation for an entire catchment. Indeed, the low pressures and high discharge modelled in the marginal areas are indicative of a well-developed, efficient subglacial system. Introducing different assumptions of k at several points in the system may provide more realistic model conditions.

4.3.4 Model performance

The subglacial drainage model in its present form has been shown to work well in the Paakitsoq context, capturing salient aspects of the melt season discharge at lake 187. The majority of the inflow (~90%) reaches lake 187 by the end of the season, a significant improvement on Long's (2008) value of ~10%. The use of smaller (1-second) time-steps (cf. Banwell *et al.,* 2010) helps to reduce issues with continuity and system blow-up. Some water remains in the system at the end of the melt season, but most of the missing 10% is lost as floodwater from junctions located in the inland sector. Further sensitivity testing is needed to investigate whether converting more of the inland conduits to a distributed configuration (i.e. conduit bundles) could increase the proportion of melt inflow reaching lake 187.

The correspondence between the measured and modelled outflow hydrographs (Figure 3.34 and 3.39) improves as the melt season progresses. In the first half of the season, the model's overestimations of outflow may be due to inaccurate constraints on the supraglacial catchments feeding the system with meltwater, or unrealistically fast routing of flow through the subglacial system. The latter could be a result of formulating parts of the drainage system as channelised when a distributed form might be more appropriate. From mid-July, the modelled and measured discharge series match well, although the model does not simulate the likely lake outburst event at JD 1500. Arnold *et al.* (1998) also found their best modelled/measured matches in the middle of the melt season, once channelisation had occurred under most of their glacier. This provides further evidence that a widespread channelised system develops at Paakitsoq over the melt season.

Modelled inflow into the system shows a realistic lag time relative to the modelled melt in the early half of the study period, as meltwater takes time to be routed through the snowpack to the relevant moulins. The lag time decreases throughout the melt season, as snow melts to reveal ice, thus enabling water to flow faster to the input moulins.

4.4 Implications for Greenland in a warming climate

4.4.1 Ice sheet dynamics

The PDD and subglacial model runs under RCPs 2.6, 4.5 and 8.5 suggest that surface melt and subglacial outflow will generally increase throughout the 21st century at Paakitsoq. However, the magnitude of this increase varies spatially across the study region, as well as across the RCPs. A warming of ~7°C over Paakitsoq by 2100 is plausible if CO_2 levels rise to three or four times their present values: under this scenario (RCP 8.5), discharge into lake 187 will be up to four times greater than at the beginning of the century, especially in the early and middle melt season (Figure 3.47). Under RCP 4.5, melt and subglacial outflow increase progressively until 2100, so discharge into lake 187 almost doubles relative to the baseline year. Under RCP 2.6, a slight decrease in temperatures by the end of the century results in lower melt volumes, and thus subglacial outflow.

It is interesting to note that the 2005 melt (Figure 3.46 and 3.47) and outflow (Figure 3.54) series regularly exceed the predicted average melt and outflow series for most of the future runs. Present inter-annual variability, therefore, is currently causing high-melt years (such as 2005) that are producing more extensive spatial distributions of low- and medium-intensity melt than average future years under some scenarios. This finding has two important implications: firstly, that western Greenland is already experiencing outflow events and resultant subglacial forcing on a scale similar to that predicted for 2050. Secondly, if the magnitude of inter-annual variability remains the same over the next century, the ice sheet at Paakitsoq could experience exceptionally high ablation and melt production over some summers in the next fifty years or so.

Since supraglacial melt is a controlling factor on the ability of a crevasse or moulin to reach the bed (Clason *et al.,* 2012), increased melt production has the potential to significantly influence the transfer of meltwater through surface-to-bed hydrologic connections in a warmer climate. Liang *et al.* (2012) have shown that during warmer years, supraglacial lakes in western Greenland drain more frequently and earlier in the melt season, extending to higher elevations. Drainage events will therefore occur more frequently over a larger area of GrIS by the end of the century (under RCP 4.5 and 8.5 at least), likely resulting in a general increase in the spatial and temporal intensity of surface-to-bed connections. This study shows that intense melting will occur in "hotspots" along the ice margin (Figure 3.49 to 3.51), which may lead to spatially differentiated changes in connections. This would result in localised impacts on subglacial network structures and basal flow, thus complicating predictions of the future state of the subglacial system at Paakitsoq.

There is much debate about whether future rises in melt input and subglacial discharge will increase (e.g. Zwally *et al.,* 2002; Shepherd *et al.,* 2009) or decrease (e.g. Bartholomew *et al.,* 2010; Sundal *et al.,* 2011) the basal sliding sensitivity in western Greenland. Both possibilities are considered here. Figure 4.2 displays the estimated seasonal movement rates above the winter baseline rate for all RCP runs, and the caption provides the methodology for these estimations. The average velocity response will likely exceed the (exceptional) 1998 movement rate observed by Zwally *et al.* (2002) by the year 2050 under all three RCPs. The movement rate will be 1.6-9.9cm/day (RCP 2.6 and 8.5 respectively) greater than the 1998 rate by the middle of the 21st century. By the end of the century, the ice at Swiss Camp will be moving faster than the winter rate by 16.3cm/day under RCP 4.5, and by 60.8cm/day under RCP 8.5. The latter is ~12x more rapid than the rate measured by Zwally *et al.* (2002) for 1996-1999. Under RCP 2.6, late-21st century ice velocity should decrease back to the 1996-1999 levels.

The estimated movement rates apply to Swiss Camp in the upper ablation zone, so ice velocity changes in the middle and lower ablation zone, where the present study is located, are expected to be even more intense. Zwally *et al.* (2002) proposed that their observed summer acceleration was caused by an increase in the water pressure at the bedrock interface, which is a well-known process in alpine glaciers (e.g. Iken *et al.,* 1983). If this mechanism is applicable to western Greenland, enhanced basal sliding under the warmer temperatures predicted by RCPs 4.5 and 8.5 may cause faster flow of ice to the margins, an increase in the thinning rate, and more rapid inward migration of the ablation zone. The incursion of a channelised system deeper into the ice sheet is therefore likely in a warmer climate. These ice dynamical changes have serious implications for GrIS's contribution to future sea-level rise (SLR): Parizek & Alley (2004) included Zwally *et al.'s* (2002) meltwater lubrication effect in a model of GrIS, reporting that a warming of 2 or 4°C by 2100 will increase net SLR by 6 or 14 cm (respectively) over the century.

In contrast to Zwally *et al.'s* (2002) mechanism, several studies (Bartholomew *et al.,* 2010; Hoffman *et al.,* 2011; Sundal *et al.,* 2011) have hypothesised that changes to the subglacial pressure under higher-melt scenarios will curtail the ice sheet velocity response. Schoof's (2010) theoretical experiments suggest that the predicted increase in mean melt supply at Paakitsoq might not necessarily lead to acceleration. Instead, channelization and subsequent glacier deceleration would occur above a critical rate of water flow, thus suppressing rather than enhancing dynamic thinning. Under Schoof's (2010) model, ice acceleration is initiated by meltwater "pulses" linked to increased rain or sudden lake drainage events (cf. Das *et al.,* 2008; Sole *et al.,* 2011).

Analysis of the pressure changes in the subglacial model provides an indication of whether Schoof's (2010) mechanism might occur at Paakitsoq. Evidence from the inland and marginal pressure curves (Figure 3.57) indicates that pressures peak earlier in the melt season as the century progresses, and as the RCP gets more extreme. Pressures also tend to reach their oscillatory state around a mean k value quicker in 2095 than in 2025. That is to say, systems that are fed with more intense melt adjust to lower pressures earlier in the melt season. As melt input increases, a given conduit will display "more channelised" behaviour faster; increased wall melting accommodates discharge more efficiently, resulting in the conduit operating at a lower steady-state pressure, for a short period at least. The basal lubrication effect of meltwater could thus be reduced (Bartholomew *et al.,* 2011).

Whilst the timings of pressure peaks and oscillations vary across runs, for the most part the eventual k value remains the same. It is also noteworthy that the k factor along the main downstream channel does not vary significantly between RCPs, even as the century progresses. The channel reaches steady state at k=0.8 within the first 500 hours (21 days) of every melt season. This might suggest that the flow speed of very marginal ice at Paakitsoq will not experience much change over the 21st century.

4.4.2 Potential contribution to sea-level rise

Ideally, future SLR should be forecast using whole-ice-sheet models accounting for all relevant processes of ice flow and mass exchange. However, since the dominant process of mass loss in Greenland's ice-terminating regions is via melt ablation (Cuffey & Paterson, 2010), the results from the present study allow for an approximate calculation of Greenland's contribution to 21st century SLR. Results presented here suggest that significant melt incursions are likely over the 21st century; the activation of previously inactive drainage networks (as hypothesised by Lewis & Smith, 2009) will draw increasing volumes of surface-derived water from the inland regions of GrIS, drastically increasing its contribution to SLR.

This study's summer DDF for ice (8.9mm^{o}C^{-1}d^{-1}) implies that melt increases by ~0.9m for every 100 PDDs. Therefore in a three-month melt season, a 1^{o}C warming increases the number of PDDs at sea level by ~100. If the melt rate averaged over Greenland's ablation zone is only half of the sea-level value (cf. Cuffey & Paterson, 2010), this implies an average increase of 0.45m yr^{-1} ^{o}C^{-1}. The ablation zone spans ~15% of the total GrIS area (2.6x10^{5}km^{2}; Cuffey & Paterson, 2010), so mass loss will total 117Gt yr^{-1} ^{o}C^{-1}, or 0.37mm yr^{-1} ^{o}C^{-1}. Assuming that the temperature increases at Paakitsoq are applicable to the whole of Greenland, a 1^{o}C warming by 2100 (predicted under RCP 2.6) would lead to net SLR of almost 2cm. A 7^{o}C warming by 2100 (predicted under RCP 8.5) would lead to net SLR of 12cm.

These mass balance estimates are illustrative, but speculative. The calculations are highly simplified and do not account for changes to the supraglacial drainage structure, snow/ice distribution or the dynamism of the ice-surface topography with time, or indeed for the predicted increase in precipitation over Greenland as the 21st century progresses (Graversen *et al.,* 2011). Moreover, the DDF for Paakitsoq is unlikely to be applicable to the whole of the GrIS ablation zone. However, as average temperatures approach or (in the case of RCP 8.5) exceed 0^{o}C at Paakitsoq, the mass loss associated with rising temperatures will offset the mass gain due to precipitation increases (Gregory & Huybrechts, 2006).

4.5 Limitations of the study

Although the 100m DEM has provided a reliable base from which to perform flow accumulation, routing and hydraulic potential calculations, the resolution of the surface and bed data may still be restricting full model accuracy. Wright *et al.* (2008) showed that subglacial water flow delineation in Antarctica is highly sensitive to small changes (~5m) in ice surface elevation, so a higher resolution DEM may be needed to account for this in the context of Paakitsoq.

Indirect evidence from sensitivity tests and inferences from pressure curves suggests that flow resistance and channel wall processes are important components of basal flow at Paakitsoq, yet the subglacial model in this study does not account for detailed subglacial physics. Englacial drainage is also highly simplified by the assumption that water drains constantly out of a lake through its moulin and reaches the bed instantly. This method does not allow for lake filling, overflowing, and episodic draining (as has been observed in western Greenland by Das et al., 2008), and negates the delaying effect of englacial percolation, particularly near the ELA (Nienow & Hubbard, 2005).

The subglacial network structure for all future runs of the subglacial model were configured using the k=0.95 catchment, which does no allow or spatio-temporal variations in k in response to varying inputs and ice topography change in a warming climate. It is assumed here that subglacial water flows along the major tributaries inferred from the flow accumulation algorithm, but this may not be valid if local factors at the bed preclude the initiation of a drainage system. Accumulation maps could be intersected with long-term average annual surface meltwater runoff patterns to identify those areas of the modelled drainage network most likely to be active (cf. Lewis & Smith, 2009).

The use of CGCM-3 in this study necessarily brings bias into calculations of future melt production. While no GCM is "incorrect", different models are based on different assumptions and parameterisations, so output for future climate scenarios may be considerably different to those employed here. Furthermore, the bias-correction used here is rather crude, in particular for precipitation, for which a constant standard deviation was assumed. A more accurate form of "local scaling" would involve analysing the actual distribution of rainfall within each individual month.

4.6 Taking the study further

Running the subglacial model in an iterative fashion, whereby the output from one run is used to reconfigure the structure and parameters of the next run, might provide more realistic spatio-temporal variations of k at Paakitsoq. It would be useful to introduce a mechanism allowing unstable distributed systems at high discharges to "switch" to a more efficient, channelised mode (Kamb *et al.,* 1985): the morphology (i.e. bundle or single) and initial diameter of each conduit could be altered after every run as a function of its average k value. Allowing for the model configuration to be adjusted as the snowline passes each moulin (cf. Arnold *et al.,* 1998; Willis *et al.,* 2002) would result in a less static simulation of the drainage system.

Intersecting this study's subglacial model more thoroughly with the supraglacial system, perhaps via a more sophisticated formulation of englacial drainage, would provide a useful perspective on future ice sheet dynamism. Colgan *et al.* (2011b) have shown that an increase in crevasse extent could result in a net decrease in basal sliding sensitivity around Paakitsoq, so it is important to quantify the proportion of crevasse-type drainage in relation to the more efficient moulin-type drainage. The effect of a warming climate on the dates of supraglacial lake drainage events at Paakitsoq could also be investigated. Lakes will develop and drain higher on the ice sheet in the future (Liang *et al.,* 2012), which has implications for the patterns of subglacial water pressure fluctuations and the inland migration of channelised drainage. The methodology of Zwally *et al.* (2002) should be developed in order to improve estimations of future ice sheet velocity and concomitant SLR.

5. Conclusions

By considering the supraglacial and subglacial hydrology of the western GrIS, this study provides insight into present-day dynamic behaviour of the ice sheet, and its potential response to future climatic warming. The subglacial drainage model replicates observed patterns of discharge at Paakitsoq well, with realistic lag times between melt production, moulin inputs, and full system outflow. Evidence from discharge, CSA and water-pressure curves suggests there is a distinctive split in "marginal" and "inland" drainage behaviour at Paakitsoq. The subglacial model physically constrains the amount of water entering the system during periods of high melt, resulting in potentially unrealistic surface overflow. The assumption that water drains straight through a pit depression rather than accumulating as a lake, and the simplified formulation for englacial drainage, may affect the model's ability to simulate the hydrology at Paakitsoq.

It is probably unrealistic to characterise the water pressure at Paakitsoq using a single k factor, but results presented here suggest k=0.925-0.95 is appropriate for modelling the catchment feeding lake 187. The accuracy of the model might be enhanced by accounting for spatio-temporal variations in k, and by changing the drainage morphology and network structure during a melt season.

The IPCC-defined trajectory RCP 8.5, which predicts an average rise of ~7°C by 2100 compared to 2010, has important implications for supraglacial melt production and subglacial drainage over the 21st century. Under this scenario, discharge into lake 187 could be up to four times greater at the end of the century than today. Present inter-annual variability is currently causing melt years that exceed the predicted average summer melt and outflow for 2095 under RCPs 2.6 and 4.5. Such variability could result in particularly intense high-melt years in future, when average temperatures will be higher than today's baseline.

As temperatures and melt volumes increase over the 21st century, larger melt discharge feeding the subglacial system will likely lead to an inland expansion of marginal drainage. It is unclear, however, whether this transition to "more channelised" drainage will increase or dampen the basal sliding mechanism identified by Zwally *et al.* (2002). By the end of the century, the ice at Swiss Camp could be moving up to 12x faster than the rate measured by Zwally *et al.* (2002) in the late 1990s. On the other hand, increased discharge may result in subglacial pressure changes that could curtail the velocity response of GrIS. The long-term effect of increased melt production on ice velocities remains unknown, but must urgently be addressed in models to improve understanding of the ice sheet's vulnerability to 21st century climate change.

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Appendices

Appendix 1

EXTRAN formulation and solution methods

EXTRAN is a pseudo-two-dimensional model because conduits can branch and converge at junctions, but the model solution only requires the slope and length of a conduit as spatial parameters, as well as its location in series relative to other conduits. The length, initial diameter, and Manning roughness are the only independent variables that must be explicitly specified. For each junction that connects to these conduits, the bed and surface elevation must be specified, as well as the junction cross-sectional area. The model calculates the conduit slope automatically based on the invert elevation of each connecting node. For each link and node in the system the primary dependent variables are discharge and hydraulic head (Roesner *et al.*, 1988).

The basic equations employed in EXTRAN are the St. Venant equations (or shallow water equations) for gradually varied, one-dimensional unsteady flow in open channels Roesner *et al.* (1988). For each link in the system, EXTRAN solves an equation that is a combination of the St. Venant momentum and continuity equations:

$$\frac{\partial Q}{\partial t} + gAS_f - 2V\frac{\partial A}{\partial t} - V^2\frac{\partial A}{\partial x} + gA\frac{\partial H}{\partial x} = 0$$

(Equation A1)

where Q is discharge through the conduit (m^3 s^{-1}), V is water velocity in the conduit (m s^{-1}), A is the cross-sectional area of flow, H is the hydraulic head (m, here invert elevation plus water depth), and S_f is the friction slope. Manning's equation gives the friction slope:

$$S_f = \frac{n^2}{AR^{4/3}} Q|V|$$

(Equation A2)

where n is Manning's roughness coefficient and R is the hydraulic radius of flow in the conduit.

Equation A1 is converted into finite difference form for the numerical solution, which employs the modified Euler (or improved polygon) method to calculate discharge in each link and head in each node for each time *t*. The values of dependent variables *Q* and *H* after each time-step Δt are calculated by projecting the values from the previous time-step across a half time-step and then a full time-step according to the slope of the finite-difference form of equation A1 (Roesner *et al.*, 1988). The basic numerical solution steps are:

1. Compute half-step discharge at $t + \Delta t/2$ in all links based on preceding full-step values of head at connecting junctions.
2. Compute half-step head at all nodes at time $t + \Delta t/2$ based on average of preceding full-step and current half-step discharges in all connecting conduits.
3. Compute full-step discharge in all links at time $t + \Delta t$ based on half-step heads at all connecting nodes.
4. Compute full-step head at time $t + \Delta t$ for all nodes based on average of preceding full-step and current full-step discharges.

Water is numerically transported through the system following this process, and discharge through the final downstream junction produces the measured outfall from the system.

Appendix 2

Subglacial flow accumulation map for 1km-resolution DEM

Subglacial catchments for 1km-resolution DEM

Appendix 3

Full model system configuration for *k*=0.95

An example of the configuration of the full system fed into the subglacial model for the *k*=0.95 catchment is given in the following tables. Conduits and their lengths (m) are listed first, with the upstream and downstream junctions. Junctions are then listed, with bedrock and surface elevation (m).

Appendix 4

Calculating the "baseline year" (1995-2004) temperature/precipitation series

<u>Temperature</u>

The mean monthly temperature average for this baseline year (from 1995 to 2004) is linearly interpolated across consecutive months, from the 15th day of one month to the 15th day of the next. An additive factor is calculated to relate the daily baseline temperature to the mean monthly average, which is then used to estimate future daily temperatures from the monthly CGCM-3 output.

<u>Precipitation</u>

The mean number of "precipitation days" (defined as days where precipitation >0mm) per month per year is calculated (see Table below). A multiplication factor is then used to estimate future daily precipitation from the CGCM-3 output: the total monthly precipitation is divided by the number of precipitation days to give the "precipitation per day of precipitation”. A random number (between 1 and the total number of days in each month) is ascribed to each day, and the highest number of days equal to the number of “precipitation days” is given the “precipitation per day of precipitation” value (e.g. for January, the top sixteen randomly-selected days are ascribed the precipitation value. The remaining fifteen are ascribed 0mm). Note that due to sporadic lack of data at the ASIAQ station, some months had to be omitted from mean precipitation calculations.

Note that it is assumed that the standard deviations from the baseline year remain the same until 2100; this assumption is necessary because the temporal resolution of CGCM-3 data precludes dispersion analysis at a daily scale.